Bank Portfolio Choice and Monetary Policy Transmission in the Face of a New Federal Funds Market

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Abstract

Recent regulatory changes shifted the relative return of holding reserves for U.S. branches of foreign banks compared to conventional domestic banks. The data show higher excess reserves held by U.S. branches of foreign banks while no decline in interbank borrowing following policy change. The paper analyzes how these reforms affected the federal funds market and how monetary policy implications depend on the trade-offs each type of bank faces. To that end, it presents an equilibrium model in the framework of Bianchi and Bigio (2014) to include two types of bank branches instead of one; foreign branches differ from domestic branches by government regulatory constraints. Domestic banks must hold deposit insurance, while foreign banks cannot. The advantage of deposit insurance is having a more stable funding source, while the disadvantage is a higher balance-sheet cost associated with reserves. In turn, it explains the observed higher excess reserves held by U.S. branches of foreign banks following a policy of interest on reserve and the consequence for bank credit supply. Interest on reserves is associated with higher reserve balances, while the deposit insurance policy will lower the price of reserves just for domestic banks. The calibration of the model finds consistent predictions for the effect of policy on the federal funds rate and the reserves held by the two types of banks. Moreover, findings suggest that U.S. branches of foreign banks are more responsive to monetary policy tools, such as the interest on reserves, because their funding source is associated with higher volatility in deposit withdrawals.

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1. Introduction

This paper studies how the implication of monetary policy depends on the heterogeneity across interbank market participants, given the evidence that suggests new policies and regulations following the financial crisis had the unintentional implication of shifting their relative return of holding reserves. The data show higher excess reserves held by U.S. branches of foreign banks while no decline in interbank borrowing following policy change. The Fed's large-scale asset purchases reduced the liquidity needed to support bank transactions. However, the interest rate on reserves allowed for arbitrage as government-sponsored enterprises (GSEs), which constituted a substantial lender in the federal funds market, was not offered a 'policy' rate before 2013.[1] The increase in reserves held by U.S. branches of foreign banks is striking. It reached over 25% of all assets in aggregate, compared to an increase of 5% from the counterpart conventional banks (Bech and Klee, 2011). All U.S. banks could exploit the arbitrage, but we observe an absolute and proportionally higher reserve increase for these foreign branches. A pressing question for effective monetary policy is how different trade-offs affecting the relative attractiveness of holding reserves influence the federal funds market and the substitution between reserves and other assets.

I study this question by expanding the model of Bianchi and Bigio (2014) of monetary policy implementation through the banking system to include two types of bank branches instead of one. The first type consists of U.S. commercial banks (from here on, domestic banks), and the second type consists of U.S. branches of foreign banks (from here on, referred to as foreign banks). Foreign banks differ from domestic banks by government regulatory constraints because domestic banks must hold deposit insurance, while foreign banks cannot. The advantage of deposit insurance is having a more stable funding source, while the disadvantage is a higher balance-sheet cost associated with reserves. The general equilibrium model exploits the new framework proposed by Bianchi and Bigio (2014),

[1] The Fed established the Overnight Reverse Repo Facility (ON RRP) to provide a floor for overnight interest rates when rates fall below the interest on reserve balances rate.

such that the mismatch of long-term assets and short-term liabilities is settled in a clearing market for withdrawals in deposits balance. Random transfers of deposits generate the reallocation of funds via the market for reserves; thereby, the aggregate overnight borrowing and lending, and the prospective valuation of loans of each type of bank, set the federal funds rate. The market conditions and the corresponding liquidity cost will dictate a bank's optimal portfolio choice. Thus, this setting allows for determining the distribution of reserves between the two banking sectors, the associated federal funds rate, alongside each sector's optimal portfolio.

In the model, government policy influences the interbank market with three monetary policy tools: interest on reserves (IOR),[2] discount window, and overnight reserve repurchasing rate (ON RRP). The first two rates are offered to banks, while the ON RRP rate is offered to GSEs. Traditional monetary policy would inflict changes in the federal funds rate by changes in the market tightness. If money is tight, overnight borrowing becomes more complex, and the corresponding higher liquidity cost will reduce bank lending. However, given a market satiated with reserves, monetary policy may change market tightness by changing the policy rates, also called the corridor rates. The theoretical literature (Hornstein, 2010; Ennis, 2014) suggests that higher IOR, for example, will increase bank reserve balances and reduce the tightness of money. The implication would be an increase in lending by banks. However, we find that lending did not increase with increases in the IOR rate during the period in question (2008-2013). Allowing market participants to differ in their valuations of overnight trades of reserves provides the theoretical grounds that higher IOR is coupled with higher tightness and contraction in aggregate lending.

The paper solves and calibrates the model with the market participants, including domestic and foreign banks, lending or borrowing reserves, and GSEs exogenously lending reserves. GSEs have historically been dominant lenders in the re-

[2]The Fed's initial policy in 2008 was to set a separate interest for excess reserves and required reserves. Historically these rates have always been equal and today defined as the interest on reserve balances or the IORB. In this paper I use the convention of calling this policy tool IOR.

serves market, a trend that increased with the increases in large-scale asset purchases and had implications for banks (Afonso et al., 2019).[3] The calibrations can match crucial features such as the amount of reserves banks choose on their portfolio and their share of lending and borrowing from the interbank market. It explains the observed higher excess reserves held by U.S. branches of foreign banks following IOR policy and the consequence for bank credit supply. The main findings suggest that although interest on reserves increases the liquidity of banks' portfolios, it has little to no effect on the market liquidity as long as GSEs are not offered the same policy rate as banks. In turn, an IOR policy change from 0 to 0.5% results in higher reserves of around 3.5 percentage points, increasing from around 6 to 9.5 percent for domestic banks and approximately 0.5 to 4 percent for foreign banks. However, since GSEs were not offered the same policy rate, the interbank market rate fell, and overnight bank borrowing of reserves increased.

The model is further calibrated to capture some of the changes in regulation that affected the federal funds market.[4] It explains the observed higher excess reserves held by U.S. branches of foreign banks following IOR policy and the consequence for bank credit supply. Interest on reserves is associated with higher reserve balances, while the deposit insurance policy will lower the price of reserves just for domestic banks. The main conclusion suggests that U.S. branches of foreign banks are more responsive to monetary policy tools, such as the interest on reserves, because their funding source is associated with higher volatility in deposit withdrawals. Possible extensions to the current model may include the direct effects of large-scale asset purchases on banks' portfolios, the federal funds market, and, therefore, the lending response.

The remainder of the paper is organized as follows. Section 1.1 includes a historical review of policy changes and the response of the two types of banks, while section 1.2 provides the literature review. Section 2 presents a comprehensive description of the model, with section 3 following the detailed equations and com-

[3]Monetary policy of large-scale asset purchases is absent from the model but is calibrated by assuming exogenous lending from GSEs.

[4]The Federal Deposit Insurance Corporation (FDIC) changed the relative return of holding reserves for U.S. branches of foreign banks compared to conventional domestic banks.

position of the model. Section 4 provides the model solution, and section 5 reports the calibration exercises. Section 6 concludes.

1.1 Historical Review

The past decade coincides with significant changes to the environment of the federal funds market; this is due, in part, to new policies and regulations following the financial crisis. For example, the Fed's large-scale asset purchases and IOR policy unintentionally shifted the interbank-market exchanges from U.S. commercial banks to mostly U.S. branches of foreign banks borrowing from GSEs. IOR was unavailable to all financial institutions, such as GSEs, and the federal funds rate fell below the IOR rate, so banks could borrow reserves low and lend high (Bech and Klee, 2011). All U.S. banks could exploit the arbitrage, but we observe a higher reserve increase for U.S. branches of foreign banks.

In addition, the Basel III Committee (Committee et al., 2010) had implications on the interbank market activity of banks. Title II of the Dodd-Frank Act changed the FDIC assessment base to include total assets less capital. The new assessment meant a balance-sheet cost or tax associated with excess reserves. Moreover, the committee required that banks hold more highly liquid assets pending bank net cash flow and leverage ratios to shift the bank's funding strategy to more stable deposits. With a more stable funding strategy, the need for precautionary reserves declines (Hein et al., 2012).

U.S. commercial banks dominantly consist of domestic with a few foreign banking institutions. These banks hold a U.S. charter and are insured and regulated by the FDIC. As of 2018, 197 branches and agencies of foreign banking organizations are not chartered and therefore not affected by the new regulations. These institutions held around 15% of all U.S. industrial and investment loans during this period. However, with higher funding costs due to the new regulations, foreign banks' activity could shift from regulated subsidiaries to unregulated U.S. branches and agencies, and the current number of branches and their importance may increase (Fillat et al. (2018), DiSalvo (2019), and Berlin et al. (2015)).

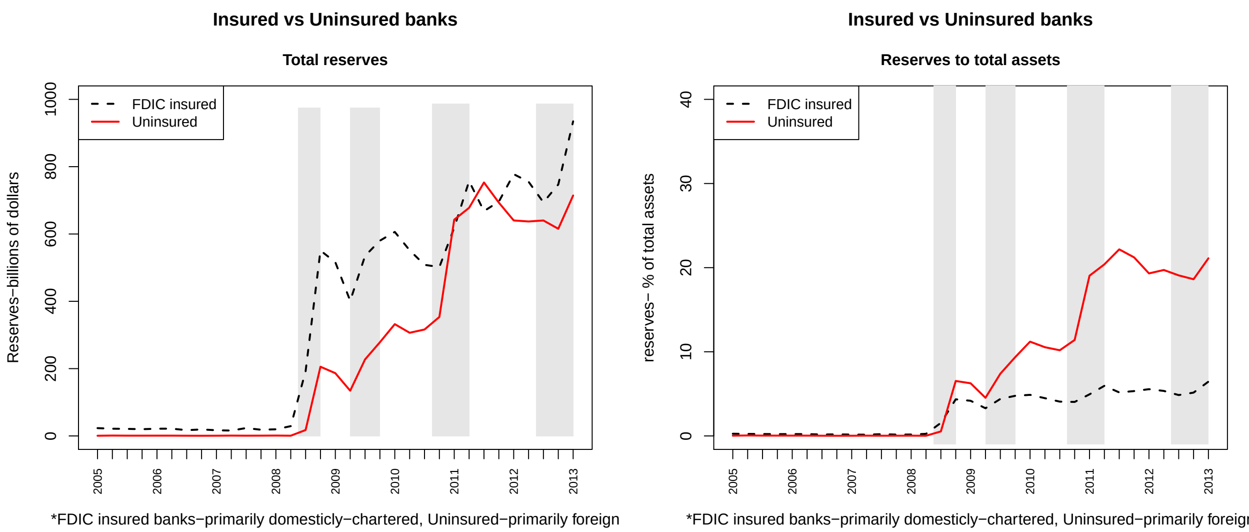


Figure 1: Reserves and Reserves to Assets of U.S. Banks

Aggregate holdings of reserves of U.S. commercial banks insured by the FDIC (insured/domestic) and U.S. branches of foreign banks uninsured by the FDIC (uninsured/foreign): Totals on the left panel and percent of all assets on the right. Shaded bars mark the central bank's Quantitative Easing with large-scale asset purchases.

Because FDIC insurance does not cover U.S. branches of foreign banks, these must manage funding solely with wholesale deposits that require significant precautionary reserves. Furthermore, The new FDIC tax associated with overnight loans implied that uninsured banks could arbitrage more efficiently than U.S. commercial (FDIC-insured) banks (Kreicher et al., 2014). Figure 1 plots the aggregate amount of reserves reported in the Financial Call Reports of commercial banking institutions.[5] The first group is of domestically chartered banks permitted to hold retail deposits. These are the U.S. commercial banks that are FDIC insured, marked with a dotted line (from here on, referred to as domestic banks). The second group of banks has no charter and, therefore, is not FDIC insured, consisting primarily of U.S. branches and agencies of foreign banks (from here on, referred to as foreign banks). The figure shows that reserves have increased dramatically

[5]Federal Financial Institutions Examination Council (FFIEC) quarterly filings. FFIEC 031, Reports of Condition and Income (also known as the Call Reports) for domestically chartered banks, and FFIEC 002, the Report of Assets and Liabilities for branches and agencies of foreign banking organizations.

in response to large-scale asset purchases, marked by the shaded bars. The aggregate amount of reserves reached $1.7 trillion by the first quarter of 2013. Domestic held 0.9 while foreign banks had 0.7 trillion dollars of reserves. Henceforth, foreign banks had acquired a considerable share of reserves compared to the size of their balance sheet. The aggregate foreign sector's reserve ratios topped 21% during this quarter. In comparison, the domestic banking sector reserves increased to around 6.5%.

Table 1: U.S. Branches of Foreign Banks Ordered by Reserve Balances

Branches or Agencies of Foreign Banking Organizations	Country	Reserves*	Assets*	% R/A**	Date
DEUTSCHE BANK AKTIENGESELLSCHAFT	GERMANY	88.71	189.40	46.8%	2011Q3
CREDIT SUISSE AG	SWITZERLAND	54.0	77.41	69.8%	2011Q1
BANK OF NOVA SCOTIA, THE	CANADA	42.74	91.66	46.6%	2011Q4
DNB BANK ASA	NORWAY	40.12	41.62	96.4%	2011Q1
SVENSKA HANDELSBANKEN AB (PUBL)	SWEDEN	34.68	36.89	94%	2011Q4
BNP PARIBAS	FRANCE	33.39	74.35	44.9%	2011Q4
SOCIETE GENERALE	FRANCE	33.13	84.67	39.1%	2011Q3
MIZUHO BANK, LTD.	JAPAN	30.56	82.01	37.3%	2011Q4
MUFG BANK, LTD.	JAPAN	28.53	95.71	29.8%	2011Q3
CANADIAN IMPERIAL BANK OF COMMERCE	CANADA	27.15	30.62	88.7%	2011Q1
SUMITOMO MITSUI BANKING CORPORATION	JAPAN	27.03	69.66	38.8%	2011Q3
ABBEY NATIONAL TREASURY SERVICES PLC	UNITED KINGDOM	26.93	27.31	98.6%	2011Q2

* Billions of dollars

**Ratio of reserves to assets

Top 12 banks with the highest reserve balances through the observed period of Q1 2005 to Q1 2013.

Table 1 lists the top foreign banks with the largest reserves on their balance sheet over the corresponding sample period. This list documents very high reserves ratios that reach 98% for some foreign banks. On the other hand, Figure 2 documents that the increase in reserves was coupled with lower interbank borrowing for the domestic sector, while not for foreign banks. The domestic foreign ratios of interbank borrowing shifted from 3 to 1 in 2008 to around 1 to 4 in 2013. Both sectors reduce interbank purchases, although domestic banks by much more.

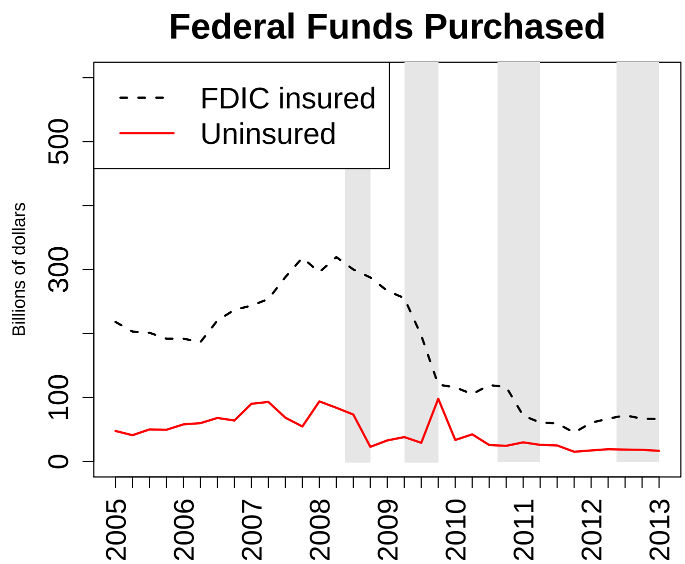


Figure 2: Federal Funds Purchases of U.S. Banks

Aggregate purchases of interbank loans by U.S. commercial banks insured by the FDIC (insured/domestic) and U.S. branches of foreign banks uninsured by the FDIC (uninsured/foreign).

Figure 3 provides the cross-sectional means of the composition of assets for domestic and foreign banks' portfolios, and Figure 4 shows the aggregate assets of the two sectors. Figure 3 further shows that after 2011, reserves captured the largest share of assets in the portfolio of foreign banks. The increase in reserves to assets was consistent through periods of large-scale asset purchases, suggesting policy changed the benefit of a foreign branch's holdings of cash reserves over other assets. These other assets marked in purple include syndicated loans on the foreign banks' portfolio of assets not reported elsewhere. On the other hand, total assets have not changed, as seen in Figure 4. These two facts imply that the increase in reserves following quantitative easing by the Fed was actually coupled with a decline in foreign bank lending resulting in a dampening of policy.

This evidence suggests that IOR coupled with the new FDIC assessment base, had increased some lenders' liquidity benefits more than others. We ask how different tradeoffs affecting the relative attractiveness of holding reserves influence

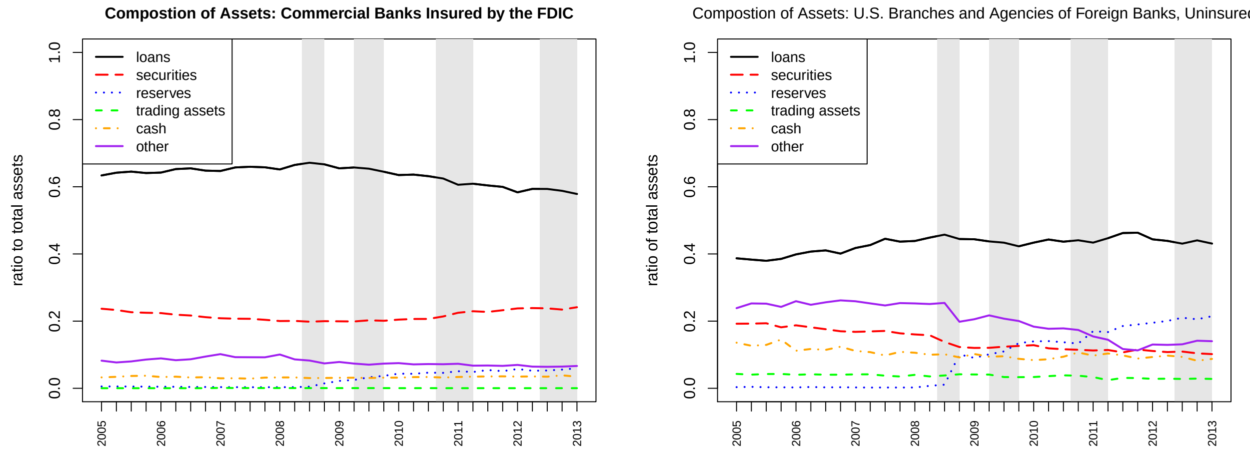


Figure 3: Composition of Assets

The cross-sectional mean of assets on portfolio of U.S. commercial banks insured by the FDIC (insured/domestic), on the left and U.S. branches of foreign banks uninsured by the FDIC (uninsured/foreign), on the right

the federal funds market and the substitution between reserves and other assets for each sector. This question is vital for effective monetary policy for two reasons. First, the policy and regulatory changes described above shifted the federal funds market trading from bank to bank trades to trades between foreign banks borrowing from GSEs. This shift has consequences for the lending channel of monetary policy because foreign banks' lending differs from domestic banks. Second, the policy tools available to influence the federal funds market rates changed because of the abundance of reserves. The rates at which banks were willing to borrow and lend were no longer governed by the scarcity of reserves but rather from the arbitrage rent that may be gained.[6] The following mode is able to account for these financial institutions' distinct tradeoffs to untwine these two distinct monetary policy implications.

[6]A good example is the Fed's introduction of an overnight reverse repurchase agreement (ON RRP) facility. The RRP operation in the interbank aimed at GSEs essentially increased the interbank rate by borrowing at the federal funds market at a higher rate than is bargained between banks and GSEs (Lester et al., 2019).

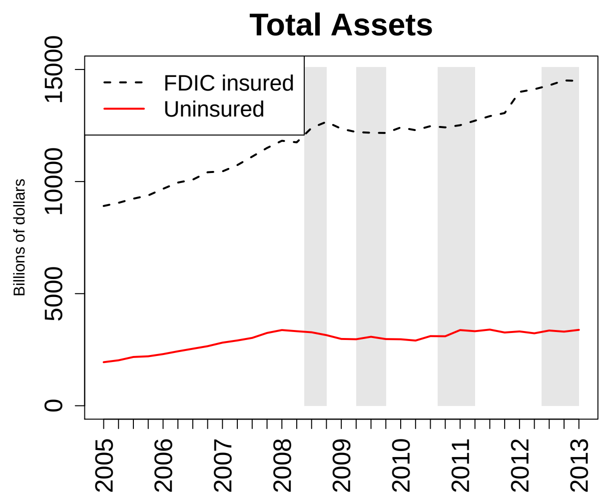


Figure 4: Federal Funds Purchases of U.S. Banks

Aggregate assets of U.S. commercial banks insured by the FDIC (insured/domestic) and U.S. branches of foreign banks uninsured by the FDIC (uninsured/foreign).

1.2 Literature Review

One of the model's assumptions is that the expansion of the FDIC assessment base affected banks' liquidity costs. Hein et al. (2012) find that the new policy increased insurance premiums to large banks that rely extensively on non-deposit funding sources. Kreicher et al. (2014) study the effect of the policy on the balance sheets of banks. They find that higher funding costs were passed on to lenders of short-term funds and lowered short-term U.S. dollar debt costs. The policy shifted funding of some domestic banks from abroad to more stable domestic deposits, while foreign banks drew considerable net wholesale funding from abroad,[7] which means that the net quantity effect of the policy is not clear.

More broadly, this paper relates to the literature on the lending channel of monetary policy. The assumption here is that monetary policy operations in the federal funds market matter for targeting the rate and the implications on banks' portfolio

[7]They measure a 44 cent increase in net due to their own office for every dollar increase in reserves at non-chartered foreign banks following the central bank's large asset purchases post the FDIC policy change, meaning that U.S. branches of foreign banks pull funds from abroad to increase their reserves account at the Fed.

choices. Monetary policy, via the lending channel, as elaborated by Bernanke and Gertler (1995), Bernanke (1990), and Kashyap and Stein (1995), can alter banks' liquidity cost and, with it, the optimal choice of reserves and loans chosen and thereby influencing broader lending conditions.

This paper also relates to the literature on the effects of interest on excess reserves such as Kashyap and Stein (2012), Hornstein (2010), Ennis (2014), Ireland (2014), and Cochrane (2014). These papers, among others, considered that banks might substitute other short-term securities for the additional interest-bearing reserves in the absence of liquidity constraints, which may imply a reduction in the overall lending activity of banks coupled with an increase in the interest rate. In which case, a new policy favoring U.S. branches of foreign banks' position in wholesale funding could magnify this problem.

Ennis et al. (2015) measure the aggregate liquidity in the banking system in response to monetary expansions of large-scale asset purchases following the financial crisis in 2008. They find that expansion efforts by the Fed in each of the three Quantitative Easing events correspond with indefinite higher liquidity so that, in aggregate, the banking system as a whole did not substitute reserves for other forms of liquid assets. Moreover, DiSalvo (2019) documents that the share of assets held by foreign banks had not changed during this period, suggesting that cash assets substitute for other assets previously held by these institutions.[8]

There is a rapidly growing stack of theoretical literature on the federal funds market. One of the first models dating back from Poole (1968) provides a static model of a bank choice of excess reserves and the federal funds market's operation. More recently, Afonso and Lagos (2015) model the federal funds market given a Poisson process dictating the interbank bargaining, for which the transaction terms determine the size of the loan and the intraday distribution of rates. Bianchi and Bigio (2017) build on this literature, alongside Atkeson et al. (2015), to transform the federal funds market model into a tractable process embedded in a dynamic general

[8]U.S. branches of foreign banks do not report all types of assets as domestic banks do. Therefore the portfolio asset composition of foreign banks reported in the next section includes other assets imputed by the data. The composition of assets over time supports the literature that suggest a substitution of reserves to other forms of non-liquid assets.

equilibrium model.

With the Fed's new conditions and complexity, a revised theoretical framework for the interbank market is on many scholars' tables. One of the first models to present these new conditions is Bech and Klee (2011). They were the first to address the federal funds rate falling below the IORB due to the presence of GSEs in a static model. Two other seminal papers include Afonso et al. (2019) and Armenter and Lester (2016). Armenter and Lester (2016) provide a model of heterogeneous banks that differ in their balance-sheet cost associated with excess reserves. In contrast, Afonso et al. (2019) focus on the impact of changes in the federal funds market's conditions for the observed distribution of reserves across banks.

The following model, I propose, builds on the OTC model of Duffie et al. (2007) by describing a market with heterogeneity in agents' valuation of an asset. It expands the Bianchi and Bigio (2014) model to include two banking sectors facing different marginal benefits of reserves. In the model, the valuation of reserves depends on a liquidity risk premium, the interbank market conditions, and the second-best overnight rates offered by the Fed. These defer across foreign and domestic banks, and hence, have implications on the optimal portfolio choice of each sector. Furthermore, each type of banks' aggregate portfolio choice matters for the outcome in the federal funds market. Hence, the foreign sector's marginal benefit of its reserves will matter per se for the domestic bank's optimal liquidity and lending choices. The steady-state solution to the model pins down the interbank rate, that is, the federal funds rate and the excess reserves of each sector.

2. Informal Description of the Model

2.1 Overview

I extend the model of Bianchi and Bigio (2014) to include the decision problem of heterogeneous interbank market participants. Their model describes the lending channel of banks via a liquidity management problem subject to the conditions in the interbank market for reserves, in which each bank is a scaled version of a bank with unit equity.

Having homogenous market participants implies that the decision of each bank is identical, but in practice, a bank's overnight lending decision differs. Moreover, the optimal choice of one type of bank depends on the relative valuation of the overnight loan (called the second-best rates) because if these rates are not identical across interbank borrowers and lenders, the optimal outcome may be to place overnight funds at the Fed for risk-free rent. For example, GSEs were not offered any policy rate prior to 2013, so they were willing to lend for a rate below the IOR rate. In which case, domestic and foreign banks could borrow low from GSEs. The model's scope allows for this interdependence of the differences across market participants to measure the impact on the lending decisions and pin down the expected federal funds rate.

I follow Bianchi and Bigio (2014) because it provides a new general equilibrium framework to capture banks' mismatch of long-term assets and short-term liabilities and its importance in implementing monetary policy through the banking system. Random transfers of deposits create a liquidity risk that determines the supply of credit and the money multiplier achieved by partitioning a bank's decision problem into a lending stage and a balancing stage. The liquidity management problem arises because lending decisions are made during the lending stage, but imbalances created by a withdrawal shock must be settled by transferring reserves from one bank to another in the balancing stage. Similar to Kashyap and Stein (1995), when cash flows constrain banks, they need to raise new funds. Here, by borrowing from other banks in the interbank market. The endogenous conditions in the interbank market determine the liquidity cost/benefit for banks and hence the demand for precautionary reserves. Thus, the expectation of the conditions in the interbank market and policy rates will quantify the bank's lending portfolio decision in the lending stage.

2.2 The Behavior of Banks

There are two types of bank branches; foreign branches differ from domestic branches by government regulatory constraints. Domestic banks must hold deposit insurance, while foreign banks cannot. The advantage of deposit insurance is having a more stable funding source, while the disadvantage is a higher balance-sheet cost associated with reserves. An additional extension includes exogenous lending of other traders (such as GSEs) that differ in their valuation of reserves.

Banks maximize utility over a stream of dividends with some risk-aversion to capture the smooth dividend distribution observed in the data. Each period is divided into a lending stage and a balancing stage. In the lending stage, risk-averse banks holding some level of equity choose the number of dividends to consume (which can be thought of as a distribution to shareholders), illiquid loans to firms, and liquid reserves, by issuing deposits from households. Because loans are illiquid at this stage, reserves serve a precautionary purpose. The random withdrawal to a bank's deposits is drawn from a common distribution across each branch. Alas, every inflow of deposits is an outflow from another bank, so the net amount across all banks equals zero, which means deposits never leave the banking sector.

Because the risk of deposits withdrawals depends on the type of bank, the associate precautionary reserves may be different. Foreign banks not insured by the FDIC raise funds from wholesale deposits associated with a more variable withdrawal shock. They are assumed to face higher withdrawal volatility, as observed empirically.[9] A domestic bank's deposits are not as volatile, but the bank must pay a Pigovian tax for its reserves that enters its utility function. This tax is calibrated to measure the FDIC assessment for its reserves holdings. In addition, GSEs place exogenous lending in the interbank market, so the imbalances created by the withdrawal shocks give rise to an interbank market of reserve transfers from one interbank agent to another. Unless stated otherwise, GSEs' second-best lending rate is zero.

[9]Hanson et al. (2015) describe the banks' portfolio design as that given deposit insurance, a bank may invest in illiquid assets to create money-like deposits because of such funds' relative stability. With no deposit insurance, a financial institution must invest in liquid assets with low fundamental risk to create non-stable deposits. Since this simplified model banks either hold illiquid loans or liquid reserves, a higher risk of withdrawals increases the demand for reserves for this sector while being silent on the substitution between reserves and other liquid assets. An interesting extension would be to include other liquid assets that could influence demand for precautionary reserves.

The interbank market, or the federal funds market, is described as an over-the-counter (OTC) market for overnight funds with Nash bargainings of bilateral trades that determine the rate for a unit of exchange. Hence the endogenous federal funds rate is the average of these rates bargained during each period. The actual price bargained depends on the market aggregates and policy interest rate. For example, if lending offers are greater than borrowing, a borrower's bargaining power is higher, lowering the rate. In contrast, if lending orders are scarce, the rate in the market will tend to be higher. This ratio of orders between aggregate borrowing and lending is called market tightness. In addition, different interbank agents face different second-best rates, determined by policy and regulation. Therefore, an interbank rate between two agents will depend on what type of borrower and lender actually meet. These probabilities are measured by the mass of orders across the types of interbank agents.

After the settlements in the market, unmatched orders are settled with second-best rates available from the Fed. Then the balancing-stage loans, reserves, interbank loans, deposits, and previous equity determines the following period equity of a bank. Lower interest gains on reserves and lower risk of holding household deposits increase the opportunity cost of domestic banks' excess reserves. Hence, a domestic bank's additional cost associated with the holdings of reserves allows foreign banks an advantage in borrowing at a lower rate than the IOR rate.[10] Lastly, the model is closed with an exogenous deposit supply and loan demand level, and the federal fund budget satisfies overnight loans not settled in the interbank market.

[10]In practice, this distortion existed before the FDIC changed its assessment base to include reserves. Arbitrage was possible since the introduction of an IOR rate because GSEs were not eligible for this interest rate, as is also documented in (Afonso et al., 2019) and (Bech and Klee, 2011).

3. The Model

3.1 Banks

The banking sector is divided into a measure of domestic banks $i \in [0, share]$ and a measure of foreign banks $j \in [share, 1]$, $share$ denoting the relative size of the domestic sector. When adding GSEs to the interbank market, $share$ still denotes the size of domestic banks in the banking sector, while $sh\bar{a}re$ denotes the share of domestic banks in the economy of interbank market participants and $\bar{a}$ the share of GSEs.

Each period $t = 0, 1, 2, ...$, is partitioned into two stages: a lending stage, denoted by $lend$, followed by a balancing stage, denoted by $balance$. The liquidity management problem arises because lending decisions are made during the lending stage, but imbalances created by a withdrawal shock must be settled by transferring reserves from one bank to another in the balancing stage. Banks cannot issue new loans in the balancing stage, so a deficit/surplus in reserves is bargained as an overnight loan contract between banks or in external markets with second-best rates settled in the next lending stage. I use the convention that $\tilde{x}_{t+1}$ denotes a portfolio variable in the lending stage and x_{t+1} denotes the end-of-period portfolio variable in the balancing stage. In what follows, D stands for a domestic sector, and F stands for the foreign sector.

The Lending Stage: In each lending stage a bank enters with some equity composed of the difference between the bank's stock of assets and liabilities. An individual bank's domestic or foreign equity are defined as follows

$$E_t^i \equiv r_t b_t^i + (r_t^{ior} - tax_t) m_t^i - r_t^{ff} m_{fft}^i - r_t^{dw} m_{dwt}^i - r_t^d d_t^i,$$

$$E_t^j \equiv r_t b_t^j + r_t^{ior} m_t^j - r_t^{ff} m_{fft}^j - r_t^{dw} m_{dwt}^j - r_t^d d_t^j,$$

with i refering to a domestic banks and j to a foreign bank. The difference is that a foreign bank's $tax = 0$.

The assets' value includes b_t loans issued at the competitively determined gross rate of r_t and m_t reserves earning the policy gross interest rate on reserve balances denoted by r_t^{ior} minus the policy determined tax denoted by tax_t when applicable. The total value of liabilities includes m_{fft} federal funds borrowed at the endogenously determined gross interbank rate r_t^{ff}, m_{dwt} discount window funds borrowed at the gross policy rate of r_t^{dw}, and d_t deposits costing the competitively determined-gross rate of r_t^d. Note that a bank may also lend federal funds. In this case, it will appear as an asset on its balance sheet.

A bank, therefore, chooses a sequence of the amount of new loans $\tilde{b}_{t+1}$, new reserves $\tilde{m}_{t+1}$, dividends denoted by c_t at the current price level P_t, and new deposits $\tilde{d}_{t+1}$, for all $t = 0, 1, 2, ...$ to maximize the expected utility function

$$\mathbb{E}\sum_{t=0}^{\infty}\frac{c_t^{1-\eta}}{1-\eta} \tag{1}$$

over a stochastic stream of dividend payments $\{c_t\}_{t=0}^{\infty}$, subject to

$$P_t c_t + \tilde{b}_{t+1} + \tilde{m}_{t+1} - \tilde{d}_{t+1} \leq E_t \tag{2}$$

a bank's budget constraint at time t,

$$\tilde{d}_{t+1} \leq \kappa\Big(\tilde{b}_{t+1} + \tilde{m}_{t+1} - \tilde{d}_{t+1}\Big) \tag{3}$$

capital requirement constraint following policy regulation parameter κ, and

$$\tilde{b}_{t+1}, \tilde{m}_{t+1}, \tilde{d}_{t+1} \;\; \geq \;\; 0 \quad \forall \;\; i \in [0, share], \;\; and \;\; j \in [share, 1] \tag{4}$$

the nonnegative constraints.

The bank's budget constraint in equation 2 specifies that a bank's value of dividends plus new assets minus liabilities at time t, must be less or equal to the the bank's current equity.

Equation 3 sets a leverage limit. It states that banks can issue deposits as long as they have enough capital collateral to back them set by an exogenous policy parameter calibrated to match regulation.[11], [12] The discount factor satisfies $0 \leq \beta \leq 1$ and $\eta \geq 0$ calibrated to measure the bank's relative risk aversion.[13]

At each time t, after lending decisions have been made, a bank will face an idiosyncratic withdrawal from its deposits in the sum of $\omega \tilde{d}_{t+1}$. The shock distribution is specific for each sector in that the foreign bank shock has a strictly higher standard deviation. $\omega_t^{Di} \sim D_t(\cdot)$ is the common distribution across all domestic banks but different from the distribution of the foreign banks $\omega_t^{Fj} \sim F_t(\cdot)$.[14] The support $[\omega_{min}, \infty)$ with $\omega_{min} \geq -1$ requires that not all funds are withdrawn at once from a specific bank.[15] The assumption that foreign banks face a high risk of withdrawal is based on the data and is expected since foreign banks only hold wholesale deposits that are different from household deposits. However, I assume all deposits remain within the banking system so that $\int_{-\infty}^{1} \omega_t^D D_t(\cdot) + \int_{-\infty}^{1} \omega_t^F F_t(\cdot) = 0$ must hold for all t.

After the withdrawal shock, a bank has some reserves surplus that equals

$$x_t^i = \left(\tilde{m}_{t+1}^i + \frac{r_{t+1}^d \omega_t^i \tilde{d}_{t+1}^i}{r_{t+1}^{ior}} \right) - \rho \tilde{d}_{t+1}^i (1 + \omega_t^i) \tag{5}$$

for domestic bank and

$$x_t^j = \left(\tilde{m}_{t+1}^j + \frac{r_{t+1}^d \omega_t^j \tilde{d}_{t+1}^j}{r_{t+1}^{ior}} \right) \tag{6}$$

[11] Lenel, Piazzesi and Schneider (2019) explicitly make reserves more valuable than loans with $\tilde{d}_{t+1}^i \leq \kappa(\varphi \tilde{b}_{t+1}^i + \tilde{m}_{t+1}^i - \tilde{d}_{t+1}^i)$, for $\varphi < 1$.

[12] Capital requirement κ is imposed on both domestic and foreign banks for two reasons. First, U.S. branches agencies of foreign banks are regulated by the country of origin that may require some capital leverage ratios. Second, this assumption makes comparison of portfolio choices of each type of bank simpler, while it does not change the main message of the results.

[13] Risk aversion is commonly used to provide the curvature needed to match dividend smoothening, as is observed in the data.

[14] Instead of defining two distributions for each type of bank that is common across each type, one could define it as a function of the bank's liquidity needs (or leverage ratio). These are observed to be significantly different across the two sectors and may result from the lack of a capital requirement on foreign banks (this will not break aggregation, as long as F_t is not a function of the bank's size).

[15] I use the same convention as in Bianchi and Bigio (2014), a reasonable assumption given the focus of this analysis is on the portfolio choice of a bank given its subjective risk of not having enough funds and not the risk of a run on the bank.

for the foreign bank, which equal to the reserves after the shock minus what is required by the central bank.[16] A negative x_t is simply a reserve deficit. $\rho \in [0, 1]$ is the reserve ratio requirement set by the monetary authority, and since foreign banks face no reserves requirement, their ρ is zero. Realizations of this surplus or deficit will determine the amount of borrowing and lending orders entered in the federal funds market.

The Federal Funds Market: The federal funds market is described by an over-the-counter (OTC) market for overnight funds where Nash bargainings of bilateral trades determine the rate for a unit of exchange; hence the endogenous federal funds rate denoted by r_t^{ff} is the average of these rates at t. Bilateral trades occur on a per-unit of reserves, each of infinitesimal size as in Atkeson et al. (2015), and more recently, in Bianchi and Bigio (2014), to ensure the tractability of a general equilibrium model. As in Duffie et al. (2007), agents' valuation of a loan depends on the type of bank because domestic and foreign banks face differences in the second-best outside lending rates set by the monetary authority. Because of this heterogeneity in agents' valuation, individual Nash-bargaining rates will depend on the type of banks engaged in a specific trade. Second-best rates include the discount window rate r_t^{dw} available for borrowing outside the market and the interest rate on reserve balances r_t^{ior} that are not lent in the market. Domestic banks incur an additional balance-sheet cost on reserves balances equal to the tax_t. In practice, the actual tax assessment depends on the bank's size. However, the tax rate is assumed to be flat for simplicity. In later sections, we add a third type of lender in the OTC market to represent GSEs with a second-best lending rate of zero.

m_b and m_l denote the marginal benefit for borrowing or lending in the market, respectively. O_t^b and O_t^l denote a specific type of bank's outside borrowing or lending option, respectively. In the Nash-bargaining problem, a bank will choose the

[16]The transfer of one unit from one bank to the next must be settled with $r_{t+1}^d / r_{t+1}^{ior}$ of reserves. Because the interest owed on the deposit at $t + 1$ must be paid by the deposit issuer, while the interest gained on reserves for that period should be retained.

bargaining rate to maximize its benefit from trade by solving

$$\max_{r_t^{ff}} \left(m_b O_t^b - m_b r_t^{ff}\right)^{\phi_t} \left(m_l r_t^{ff} - m_l O_t^l\right)^{1-\phi_t},$$

with $\phi_t \leq 1$ the bargaining power variable at time t. Solving for the first-order conditions, we get that

$$r_t^{ff} = (1-\phi_t)O_t^b + \phi_t O_t^l. \tag{7}$$

Hence a rate bargained in the federal funds market must fall in the range of $\left[r_t^{ior} - tax_t, r_t^{dw}\right]$. The actual rate will depend on the market conditions described by ϕ_t.

A bank will place borrowing or lending orders based on the realization of x_{t+1} and the existence of a spread between borrowing in one market and lending in another. For example, when $\mathbb{E}(r_t^{ff}) < r_t^{ior}$, the existing arbitrage implies a foreign bank will place borrowing orders even if it has excess reserves. The different possible cases are described below with more detail in Appendix C. Only one round of bargaining is executed for each dollar of exchange so that $\mathbb{E}(r_t^{ff})$ does not change across multiple cycles of trades, essentially simplifying the analysis.

A match in the market can occur between lending and borrowing orders randomly matched according to the relative mass of borrowing orders to lending orders called the market tightness variables denoted by θ_t and $\bar{\theta}_t$. Denoting M_t^- and M_t^+ the mass of borrowing and lending orders respectfully, $\theta_t = M_t^-/M_t^+$ is the market tightness before trades have accrued, and $\bar{\theta}_t$ is the tightness accounting for matching market frictions based on a Passion probability function of arrival times, such that

$$\bar{\theta}_t = \begin{Bmatrix} 1 + (1+e^{\lambda})(\theta_t - 1) & \text{if } \theta_t > 1 \\ (1 + (\theta_t^{-1} - 1)e^{\lambda})^{-(e^{-\lambda} + \bar{\phi})} & \text{otherwise,} \end{Bmatrix} \tag{8}$$

with λ the market friction parameter for the Poisson probability function's arrival rate of matches calibrated to match the observed market tightness. The endogenous bargaining weights ϕ_t depends on this tightness, such that

$$\phi_t = \begin{cases} \left(\left(\frac{\bar{\theta}_t}{\theta_t}\right)^{\bar{\phi}} - 1\right)\frac{\theta_t}{\theta_t - 1}(e^{\lambda} - 1)^{-1} & \text{if } \theta_t > 1 \\ \bar{\phi} & \text{if } \theta_t = 1 \\ \left(\frac{1}{1-\theta_t}\right)\left(\left(\frac{\bar{\theta}_t}{\theta_t}\right)^{\bar{\phi}} - 1\right)(e^{\lambda} - 1)^{-1}(\bar{\phi} + e^{\bar{\phi}-\lambda}) & \text{otherwise.} \end{cases} \quad (9)$$

A parameter of $\bar{\phi} = 0.5$ implies an equal bargaining parameter when the lending orders and borrowing orders are equal. These functions allow for bargaining weights that are exponential with market tightness as suggested in Bianchi and Bigio (2017).

The market tightness and the friction parameter λ determine the probability of matching lending orders and borrowing orders, such that

$$\gamma_t^+ = \begin{cases} 1 - e^{-\lambda} & \text{if } \theta_t \geq 1 \\ \theta_t(1 - e^{-\lambda}) & \text{otherwise,} \end{cases} \quad (10)$$

and

$$\gamma_t^- = \begin{cases} 1 - e^{-\lambda} & \text{if } \theta \leq 1 \\ \theta_t^{-1}(1 - e^{-\lambda}) & \text{otherwise.} \end{cases} \quad (11)$$

In this form, the probabilities of γ_t^+ and γ_t^- describe two Poisson distributions of arrival times with the rate of arrivals set by λ. For example, if $\theta_t \geq 1$, total borrowing orders exceed the total of lending orders, the probability of matching lending order γ_t^+ only depends on the market friction λ for a match to occur. If, however, $\theta_t < 1$, this probability of a match is scaled by θ_t. The analog for γ_t^- implies that $\theta_t \leq 1$ described the case where the mass of lending orders exceeds borrowing orders.

Since the bargaining of an interbank rate depends on each side's type, we need to keep track of each bank's mass of borrowing and lending orders (foreign or domestic). $I_t^- \in [0, M_t^-]$ denotes the mass of domestic borrowing, and $J_t^- = M_t^- - I_t^-$ foreign borrowing. Similarly, $I_t^+ \in [0, M_t^+]$ and $J_t^+ = M_t^+ - I_t^+$ denotes the mass of domestic and foreign lending. Hence, the conditional probability of a borrowing order matched with a domestic lending order is given by

$$\gamma_{Dt}^{-} = I_t^{-}/M_t^{-}. \tag{12}$$

The conditional probability that a lending order is matched with a domestic bank is given by

$$\gamma_{Dt}^{+} = I_t^{+}/M_t^{+} \tag{13}$$

-the analog probabilities of meeting a foreign bank equal

$$(1 - \gamma_{Dt}^{-}) = (M_t^{-} - I_t^{-})/M_t^{-}, \tag{14}$$

and

$$(1 - \gamma_{Dt}^{+}) = (M_t^{+} - I_t^{+})/M_t^{+}. \tag{15}$$

s_t denotes the amount each bank puts in the market. Although a bank may not place lending orders beyond its excess reserves, placing borrowing orders above the deficit addresses the possibility that borrowing reserves at the interbank market occur for reasons beyond liquidity constraints.[17] A limit (or satiation) for borrowing orders placed above a deficit is necessary to ensure a unique solution to the model. To limit this amount, define the maximum share of borrowing orders placed while still in excess to equal the current reserves on hand (i.e., $s_t^j = -x_t^j$). The unmatched amount of s_t will trade in the Second-best market.

With the above structure of the market, we can characterize the liquidity yield for banks. A domestic bank, with a shock $\omega_t^i > \omega^{*i}$, has a reserves surplus. If matched in the interbank market, it will obtain a return of r_t^{ff} or otherwise $r_t^{ior} - tax_t$. We need that $r_t^{ff} \geq (r_t^{ior} - tax_t)$ for the market to exist, hence

$$s_t^i(\omega_t^i | \omega_t^i > \omega_{i\,t}^{*}) = x_t^i;$$

[17]Having lending orders beyond excess is senseless if these cannot be fulfilled in the current time period.

all excess reserves are placed in the market. In equilibrium, only a fraction of γ_t^+ is matched and earns the return of r_t^{ff}. The unmatched funds earn the net rate of $(r_t^{ior} - tax_t)$. The resulting liquidity yield is the net gain of lending funds in the interbank market and follows

$$\chi_{Dt}^{+} = \gamma_t^{+}[\overset{D+}{\mathbb{E}}(r_t^{ff}) - (r_t^{ior} - tax_t)],$$

with $\mathbb{E}^{D+}(r_t^{ff}) = (1-\phi_t)r_t^{dw} + \phi_t(r_t^{ior} - tax_t)$, defined as the expected federal funds rate for domestic lenders over the expectation of the type of borrower they meet at the market and their respective outside rates. For the domestic lender the expected federal funds rate is simply the Nash-bargaining weighted average of its outside lending rate and the outside borrowing rate which in this case is equivalent across domestic and foreign borrowers. The Nash-bargaining, ϕ_t, is endogenously set by the mass of borrowing to lending orders on each side of the market.[18]

Similarly, the condition $r_t^{ff} \leq r_t^{dw}$ requires that domestic banks with a reserve deficit first place borrowing orders in the market, i.e.,

$$s_t^i(\omega_t^i | \omega_t^i < \omega_{i\,t}^*) = x_t^i.$$

γ_t^- is the probability orders are matched, and the rest is borrowed from the central bank. The corresponding liquidity cost of a reserve deficit for a domestic bank is

$$\chi_{Dt}^{-} = \gamma_t^{-}[\overset{D-}{\mathbb{E}}(r_t^{ff}) - (r_t^{ior} - tax_t)] + (1-\gamma_t^{-})[r_t^{dw} - (r_t^{ior} - tax_t)],$$

with $\mathbb{E}^{D-}(r_t^{ff}) = (1-\phi_t)r_t^{dw} + \phi_t(\gamma_{Dt}^{-}(r_t^{ior} - tax_t) + (1-\gamma_{Dt}^{-})r_t^{ior})$. Here the expected market rate depends on the probability of matching the borrowing order with either a domestic lender or a foreign lender since the second-best rates of the two are different. The liquidity cost for domestic banks is summarized as a function of

[18]For more details on how ϕ_t is calculated I refer the interested reader to Bianchi and Bigio (2017).

the surplus x_t^i and is equal to

$$\chi_t^D(s_t(x_t)) = \begin{Bmatrix} \chi_{Dt}^{+} s_t^i & \text{if } x_t \leq 0 \\ \chi_{Dt}^{-} s_t^i & \text{otherwise.} \end{Bmatrix} \tag{16}$$

A foreign bank, facing a shock $\omega_t^j > \omega_j^*$, can either lend to another bank at the federal funds rate or lend to the central bank to gain r_t^{ior} for each dollar of reserves. When the market conditions are such that r_t^{ior} falls below r_t^{ff}, the bank will place lending orders for every dollar in excess in the federal funds market. However, if $r_t^{ior} - tax_t \leq r_t^{ff} \leq r_t^{ior}$, a rate still within the domestic bank's boundaries to place lending orders, a foreign bank will place borrowing orders despite excess reserves. The liquidity benefit of having excess reserves depends on if the foreign bank is a lender or an arbitrageur.[19] Therefore, a lender's expected return on lending is

$$lender_t = \gamma_t^{+}(\overset{F+}{\mathbb{E}}(r_t^{ff}) - r_t^{ior}),$$

with the expected federal funds rate equal to $\mathbb{E}^{F+}(r_t^{ff}) = (1-\phi_t)r_t^{dw} + \phi_t r_t^{ior}$, depending on the lender's expectation of meeting a domestic or foreign borrower given the same second-best borrowing rate, r_t^{dw}. The arbitrage of borrowing from the federal funds market and lending to the central bank is equal to

$$arbitrage_t = \gamma_t^{-}(r_t^{ior} - \overset{ab}{\mathbb{E}}(r_t^{ff})),$$

with the arbitrageur's expected federal funds rate equal to $\mathbb{E}^{ab}(r_t^{ff}) = (1-\phi_t)r_t^{ior} + \phi_t(\gamma_{Dt}^{-}(r_t^{ior} - tax_t) + (1-\gamma_{Dt}^{-})r_t^{ior})$ that depends on the arbitrage's expectation of either meeting a domestic or foreign lender given the known relative mass of the two. A foreign bank will choose to arbitrage if $lend_t < arbitrage_t$, otherwise it will lend its excess reserves in the market, which means that a foreign bank with excess reserves may place funds in either side of the market, depending on the expected

[19]The presence of GSEs presents a similar condition for domestic banks as well, with the rate falling below r_t^{ior} given GSEs' second-best rate is zero. This possibility is revisited in the following sections with the extension of the model to include GSEs.

return. Hence

$$s_t^j(\omega_t^j|\omega_t^j > \omega_j^*) = \begin{Bmatrix} x_t^j & \text{if } lend_t \geq arbitrage_t \\ -x_t^j & \text{otherwise} \end{Bmatrix}$$

and the liquidity benefit equals

$$\chi_{Ft}^+ = \begin{Bmatrix} \gamma_t^+(\overset{F+}{\mathbb{E}}(r_t^{ff}) - r_t^{ior}) & \text{if } lend_t \geq arbitrage_t \\ \gamma_t^-(r_t^{ior} - \overset{ab}{\mathbb{E}}(r_t^{ff})) & \text{otherwise.} \end{Bmatrix}$$

Lastly, a foreign bank with shock $\omega^j < \omega_j^*$ has a reserves deficit. In which case, the condition

$$s_t^j(\omega_t^j|\omega_t^j < \omega_{jt}^*) = x_t^i$$

states that borrowing orders are first placed in the interbank market because the discount window rate is assumed to be higher than the expected federal funds rate. The expected cost of borrowing is

$$\chi_{Ft}^- = \gamma_t^-[\overset{F-}{\mathbb{E}}(r_t^{ff}) - (r_t^{ior} - tax_t)] + (1-\gamma_t^-)[r_t^{dw} - r_t^{ior}],$$

where $\mathbb{E}^{F-}(r_t^{ff}) = (1-\phi_t)r_t^{dw} + \phi_t(\gamma_{Dt}^-(r_t^{ior} - tax_t) + (1-\gamma_{Dt}^-)r_t^{ior})$, and the foreign bank's liquidity cost as a function of the surplus x_t^j equals

$$\chi_t^F(s_t(x_t)) = \begin{Bmatrix} \chi_{Ft}^+ s_t^j & \text{if } x_t \leq 0 \\ \chi_{Ft}^- s_t^j & \text{otherwise.} \end{Bmatrix} \tag{17}$$

The difference between borrowers' and lenders' liquidity cost creates an endogenous wedge between a surplus' marginal value and a deficit's marginal cost. This wedge results from the fiction of entering the interbank market. Frictions that depend on a market's liquidity pending domestic and foreign banks' lending and borrowing decisions, which means that even with an abundance of reserves that fulfill the liquidity needs of domestic banks, the existence of arbitrageurs may result in a tight market. Hence the optimal portfolio of domestic and foreign banks depends on the spread between each sector's outside rates, and the distribution of reserves across them and the endogenous federal funds rate will equal the weighted average of all rates bargained in the market, such that

$$r_t^{ff} = \begin{cases} \gamma_t^+ \gamma_{tD}^+ \overset{D+}{\mathbb{E}}(r_t^{ff}) + \gamma_t^- \gamma_{Dt}^- \overset{D-}{\mathbb{E}}(r_t^{ff}) + \gamma_t^+ \gamma_{Ft}^+ \overset{F+}{\mathbb{E}}(r_t^{ff}) + \gamma_t^- \gamma_{Ft}^- \overset{F-}{\mathbb{E}}(r_t^{ff}) & \text{if } lend_t \geq arbitrage_t \\ \gamma_t^+ \gamma_{tD}^+ \overset{D+}{\mathbb{E}}(r_t^{ff}) + \gamma_t^- \gamma_{tD}^- \overset{D-}{\mathbb{E}}(r_t^{ff}) + \gamma_t^- \gamma_{tF}^+ \overset{ab}{\mathbb{E}}(r_t^{ff}) + \gamma_t^- \gamma_{tF}^- \overset{F-}{\mathbb{E}}(r_t^{ff}) & \text{otherwise.} \end{cases}$$

The Balancing Stage: Following a shock ω_t to deposits domestic banks' end-of-balancing-stage deposits equal to

$$d_{t+1}^i = (1 + \omega_t^{Di})\tilde{d}_{t+1}^i, \tag{18}$$

loans are illiquid so that domestic banks' end-of-balancing-stage loans equal

$$b_{t+1}^i = \tilde{b}_{t+1}^i. \tag{19}$$

Following the bargaining problem that depends on the probabilities of a match and the number of orders placed on each side of the interbank market, we get the end-of-balancing-stage reserves

$$m_{t+1}^i = \tilde{m}_{t+1}^i + \frac{r_{t+1}^d \omega_t^{Di} \tilde{d}_{t+1}^i}{r_{t+1}^{ior}} + m_{fft+1}^i + m_{dwt+1}^i. \tag{20}$$

Equation 20 states that the end-of-balancing-stage reserves amount to the reserves following the withdrawal shock plus any additional reserves borrowed from either the interbank market, denoted by m_{fft+1}^i, or from the central bank's discount window, denoted by m_{dwt+1}^i. Given the surplus for a domestic bank specified in 5, these overnight reserves borrowed equal to

$$(m_{fft+1}^i, m_{dwt+1}^i) = \begin{cases} (-x_t^i \gamma_t^-, -x_t^i(1-\gamma_t^-)) & \text{for } x_t^i < 0 \\ (-x_t^i \gamma_t^+, 0) & \text{for } x_t \geq 0, \end{cases} \tag{21}$$

so that a negative m_{fft+1}^i is an overnight loan by the bank to another bank, and

$m^i_{dw\,t+1} \geq 0$.

A foreign bank end-of-balancing-stage portfolio shares will follow

$$d^j_{t+1} = (1+\omega_t)\tilde{d}^j_{t+1}, \tag{22}$$

$$b^j_{t+1} = \tilde{b}^j_{t+1}, \tag{23}$$

and

$$m^j_{t+1} = \tilde{m}^j_{t+1} + \frac{r^d_{t+1}\omega^{Fj}_t \tilde{d}^j_{t+1}}{r^{ior}_{t+1}} + m^j_{ff\,t+1} + m^j_{dw\,t+1}. \tag{24}$$

Equations 22 and 23 are the end-of-balancing-stage deposits and loans, and equation 24 is the end-of-balancing-stage reserves that amounts to the reserves after the withdrawal shock plus any additional reserves borrowed from either the interbank market or discount window. Given the surplus for a foreign bank specified in equation 6, these overnight reserves borrowed and lent equal to

$$(m^j_{ff\,t+1}, m^j_{dw\,t+1}) = \begin{cases} (-x^j_t\gamma^-_t, -x^j_t(1-\gamma^-_t) & \text{for } x^j_t < 0 \\ (-x^j_t\gamma^+_t, 0) & \text{if } lend_t \geq arbitrage_t \text{ for } x^j_t \geq 0 \\ (x^j_t\gamma^-_t, 0) & \text{if } lend_t < arbitrage_t \text{ for } x^j_t \geq 0, \end{cases} \tag{25}$$

and depend on if the foreign bank with excess reserves finds it optimal to lend its excess reserves or arbitrage by borrowing against them.

With the assumption of no aggregated risk, the rest of the aggregates follow $E^D_{t+1} \equiv \int_i E^i_{t+1}di$, $E^F_{t+1} \equiv \int_j E^j_{t+1}dj$, $B^D_{t+1} \equiv \int_i b^i_{t+1}di$, $B^F_{t+1} \equiv \int_j b^j_{t+1}dj$, $D^D_{t+1} \equiv \int_i d^i_{t+1}di$, $D^F_{t+1} \equiv \int_j d^j_{t+1}dj$, $M^D_{t+1} \equiv \int_i m^i_{t+1}di$, $M^F_{t+1} \equiv \int_j m^j_{t+1}dj$, $X^D_{t+1} \equiv \int_i m^{ff}_{t+1}{}^i di$, $X^F_{t+1} \equiv \int_j m^{ff}_{t+1}{}^j dj$, $DW^D_t \equiv \int_i m^i_{dwt}di$, $DW^F_t \equiv \int_j m^j_{dwt}dj$. These are the aggregate equity, loans, deposits, reserves, interbank balances, and discount window loans for the aggregates across each type of bank, respectively.

3.2 Monetary Policy, Loan Demand , and Deposit Supply

The monetary authority has a simple balance sheet of reserves denoted by M^s_t, and discount window loans to domestic and foreign banks denoted by DW^D_t and

DW_t^F respectively. The monetary authority sets the corridor rates $\{r_t^{ior}, r_t^{dw}\}$ to influence banks' portfolio decisions via changes in the banks' expected liquidity cost function.[20] In the lending stage the budget of the monetary authority must satisfy

$$r_t^{ior} M_t^s - r_t^{dw}(DW_t^D + DW_t^F) \leq \tilde{M}_{t+1}^s. \tag{26}$$

Stating that the lending-stage reserves will depend on last period reserves with interest net the interest retained from discount window loans. The balancing-stage reserves are

$$M_{t+1}^s = \tilde{M}_{t+1}^s + (DW_{t+1}^D + DW_{t+1}^F), \tag{27}$$

and by substitution, the monetary authority budget constraint becomes

$$r_t^{ior} M_t^s - r_t^{dw}(DW_t^D + DW_t^F) \leq M_{t+1}^s - (DW_{t+1}^D + DW_{t+1}^F). \tag{28}$$

Since policy directly affects both foreign and domestic banks' interbank lending decisions, it may also indirectly affect the targeted federal funds rate given the interaction of the two types of agents in the market. For example, in a situation in which the expected federal funds rate is lower than the interest on reserve balances, an increase in funds may still result in a tight federal funds market because of the presence of arbitrageurs. In which case, the policy is dampened by their presence.

Lastly, loan demand and deposit supply depend on the relevant market rates and the exogenous semi-elasticity of credit demand ϵ and the semi-elasticity of deposit supply υ, set strictly greater than zero. Market-clearing equilibrium loans and deposits equate with

$$B_{t+1}^d = \left(\frac{\Theta_t^b}{r_{t+1}^b}\right)^{\epsilon}, \tag{29}$$

[20]The monetary authority can also influence the market by changes to the reserves available with open market operations, and may issue private sector loans that capture unconventional monetary policy. This extension is out of the scoop of this paper addressing the steady-state solutions of a policy change.

and

$$D^s_{t+1} = \left(\frac{\Theta^d_t}{r^d_{t+1}}\right)^{-\upsilon}, \tag{30}$$

such that the steady-state conditions determine the intercepts Θ^b_t and Θ^d_t.

3.3 Definition of the Competitive Equilibrium and Market Clearing Conditions

The competitive equilibrium is defined given the initial sequence of the distribution of $\{d^i_0, d^j_0, b^i_0, b^j_0, m^i_{ff0}, m^j_{ff0}, m^i_{dw0}, m^j_{dw0}\}$ across banks, the deterministic sequence of government policy variables $\{\rho_t, M^s_t, \kappa, r^{ior}_t, r^{dw}_t\}_{t\geq 0}$, the set of deterministic sequence of real prices $\{r^b_t, r^d_t\}_{t\geq 0}$, the deterministic sequence of aggregate variables $\{D_{t+1}, B_{t+1}, M_{t+1}, DW^D_{t+1}, DW^F_{t+1}, X^D_{t+1}, X^F_{t+1}\}$, the stochastic sequence of matching probabilities and the federal funds rate $\{\gamma^+_t, \gamma^-_t, \gamma^+_{Dt}, \gamma^-_{Dt}, r^{ff}_t\}_{t\geq 0}$, and a stochastic sequence of banks' policy variables $\{\tilde{b}^i_{t+1}, \tilde{m}^i_{t+1}, \tilde{d}^i_{t+1}, c^i_t, m^i_{fft}, m^i_{dwt}, \tilde{b}^j_{t+1}, \tilde{m}^j_{t+1}, \tilde{d}^j_{t+1}, c^j_t, m^j_{fft}, m^j_{dwt}\}_{t\geq 0}$, such that

- The variables $\{\tilde{b}^i_{t+1}, \tilde{m}^i_{t+1}, \tilde{d}^i_{t+1}, c^i_t, \}$ solve the domestic bank's problem.
- The variables $\{\tilde{b}^j_{t+1}, \tilde{m}^j_{t+1}, \tilde{d}^j_{t+1} c^j_t, \}$ solve the foreign bank's problem.
- $\{m^i_{fft}, m^i_{dw}, m^j_{fft}, m^j_{dwt}\}$ are given by the conditions in the federal funds market, monetary policy, and the realized shock to deposits.
- The central bank budget constraint given in (28) is satisfies
- Aggregate loans are consistent with the exogenous demand for loans given by (29), and aggregate deposits are consistent with the exogenous supply of deposits given by (30)
- For all $t \geq 0$ the market clearing conditions are satisfied.

$$B^D_{t+1} + B^F_{t+1} = B^d_{t+1} \tag{loan market}$$

$$D^D_{t+1} + D^F_{t+1} = D^s_{t+1} \tag{deposit market}$$

$$M^D_{t+1} + M^F_{t+1} = M^s_{t+1} \qquad \text{(reserves market)}$$

$$\int_i m^i_{fft+1} di + \int_j m^j_{fft+1} dj = 0 \qquad \text{(interbank market)}$$

- The matching probabilities $\{\gamma^+_t, \gamma^-_t, \gamma^+_{Dt}, \gamma^-_{Dt}\}$ and r^{ff}_t are consistent with the aggregate surplus and deficit $\{M^-_t, M^+_t, I^+_t, I^-_t\}$ as is given by (10), (11),(12), and (13).

3.4 Government-sponsored agencies (GSEs)

Given the presence of GSEs in the market, we would like to address the implications for interbank orders and banks' optimal portfolios. One straightforward extension is to add an exogenous level of interbank lending orders from GSEs introduced to the interbank market at each stage of the optimization so that the level of GSEs' lending stays constant through iterations of the banking-side choice. The endogenous federal funds rate is also influenced by the presence of GSEs with an outside lending option of r^{rrp}_t, the Overnight Reverse Repo Facility (ON RRP) rate. For this extension, we define shares of lending orders from each sector so that the mass of lending orders becomes $M^+_t = \bar{a}G^+ + sh\bar{a}reD^+_t + (1 - sh\bar{a}re)F^+_t$. Here D^+_t and F^+_t are the lending order of the representative domestic and foreign banks, and G^+ is the exogenous amount of GSEs' interbank lending orders. The share of each lender in the aggregate market equals to $sh\bar{a}re$, $1 - sh\bar{a}re$, and $\bar{a}$, respectively. In addition, the interbank market clearing condition becomes

$$\gamma^+_t(sh\bar{a}reD^+_t + \bar{a}G^+ + (1 - \bar{a} - sh\bar{a}re)F^+_t = \gamma^-_t(shareD^-_t + (1 - share)F^-_t). \tag{31}$$

From this formulation we can adjust the probabilities of matching each order with either type of lender and the corresponding expected federal funds rates given the outside option for a GSE equals r^{rrp}.

4. Model Solution

4.1 Solving the Banking Model

The model closely follows the related Bianchi and Bigio model (denoted as the BB model). Rather than repeating their seminal work, I describe some features of the extended domestic and foreign banking model (denoted by DF model) in the model's solution and refer the reader for details for its derivation to Bianchi and Bigio (2014).

Adding foreign banks to the model entails significant differences in the steady-state solution and conclusion for the bank optimal portfolio choice. The main differences between the two banking sectors are the value of reserves in the budget constraint (the balance-sheet cost) and each sector's distribution of the idiosyncratic shock ω, as observed in the data. The latter assumption supports the idea that U.S. branches of foreign banks uninsured wholesale deposits expose the bank to higher risk. In the steady-state results to follow, we find that this higher risk increases foreign banks' excess reserves as expected. However, it also changes domestic banks' optimal portfolio choice of reserves, albeit the relatively small share of the foreign sector to the domestic sector because foreign banks' increase in precautionary reserves reduces the interbank market tightness and thereby lowers the liquidity cost. We can see the reasoning in the model's solution.

Let $\overset{lend}{V_t^i}(\cdot)$ define the value function of a domestic bank during the lending stage and $\overset{balance}{V_t^i}(\cdot)$ the value function of a domestic bank during the balancing stage. The lending-stage-stochastic problem can be stated recursively as

$$\overset{lend}{V_t^i}(E_t^i) = \max_{\{\tilde{b}_{t+1}^i, \tilde{m}_{t+1}^i, \tilde{d}_{t+1}^i, c_t^i\}} u(c_t^i) + \mathbb{E}_{\omega^i, \omega^j}[\overset{balance}{V_t^i}(\tilde{b}_{t+1}^i, \tilde{m}_{t+1}^i, \tilde{d}_{t+1}^i, \omega_t^i)],$$

subject to the budget, capital, and non-negative constraints in equations (2), (3), and the non-negativity constraints in (4), respectively, given bank preferences described in equation (1). Then in the balancing stage, the decision problem of the domestic banks summarized recursively follows

$$\overset{balance}{V_t^i}(\tilde{d}^i_{t+1}, \tilde{b}^i_{t+1}, \tilde{m}^i_{t+1}, \omega^i_t) = \beta \overset{lend}{V^i_{t+1}}\left(d^i_{t+1}, b^i_{t+1}, m^i_{t+1}, m^i_{fft+1}, m^i_{dwt+1} | \theta_t\right),$$

given (18), (19), (5), (21), and (20). With the definition for equity, a substitution of the balancing-stage conditions into the end-of-stage variables arrives at

$$E^i_{t+1} = r_{t+1}\tilde{b}^i_{t+1} + (r^{ior}_{t+1} - tax_t)\tilde{m}^i_{t+1} - r^d_{t+1}\tilde{d}^i_{t+1} + \chi^D_{t+1}(\tilde{m}^i_{t+1}, \tilde{d}^i_{t+1}, \omega^i_t | \theta_t) \tag{32}$$

By substituting the above into the lending-stage value function, we get

$$\overset{lend}{V_t^i}(E^i_t) = \max_{\{\tilde{b}^i_{t+1}, \tilde{m}^i_{t+1}, \tilde{d}^i_{t+1}, c^i_t\}} u(c^i_t) + \mathop{\mathbb{E}}_{\omega^i, \omega^j}[\beta \overset{lend}{V^i_{t+1}}(E^i_{t+1} | \theta_t)]$$

subject to (2), (3), (32), and (4). θ_t is endogenously determined by the aggregate ratio of borrowing to lending orders in the federal funds market and depends on both banking sectors' optimal decisions.

Similarly, defining $\overset{lend}{V_t^j}(\cdot)$ and $\overset{balance}{V_t^j}(\cdot)$ as the value function of a foreign bank during the lending and balancing stage, we arrive at the single-stage-stochastic recursive problem of

$$\overset{lend}{V_t^j}(E^j_t) = \max_{\{\tilde{b}^j_{t+1}, \tilde{m}^j_{t+1}, \tilde{d}^j_{t+1}, c^j_t\}} u(c^j_t) + \mathop{\mathbb{E}}_{\omega^i, \omega^j}[\beta \overset{lend}{V^j_{t+1}}(E^j_{t+1} | \theta_t)],$$

subject to (2), (3), (4), and

$$E^j_{t+1} = r_{t+1}\tilde{b}^j_{t+1} + r^{ior}_{t+1}\tilde{m}^j_{t+1} - r^d_{t+1}\tilde{d}^j_{t+1} + \chi^F_{t+1}(\tilde{m}^j_{t+1}, \tilde{d}^j_{t+1}, \omega^j_t | \theta_t) \tag{33}$$

This result follows since once the withdrawal shock is realized, the bank's choice is already made. All that matters in the lending stage is the expectations of the shock and the liquidity cost function associated with such shock. There is no maximization in the balancing stage, rather a deterministic end-of-stage portfolio of a bank given the aggregate market conditions for liquid funds.

Bianchi and Bigio (2014) further show that given the homogeneity of the utility function in η, we have that $V_t(E_t) = v_t E_t^{1-\eta} - \frac{1}{(1-\beta)(1-\eta)}$, for some function v_t. It follows that the maximization problem equals

$$v_t E_t^{1-\eta} - \frac{1}{(1-\beta)(1-\eta)} = \max_{\{\tilde{b}_{t+1}, \tilde{m}_{t+1}, \tilde{d}_{t+1}, c_t\}} \frac{c_t^{1-\eta}}{1-\eta} + \mathbb{E}_{\omega}\left[\beta v_{t+1} E_{t+1}^{1-\eta} - \frac{1}{(1-\beta)(1-\eta)}\right],$$

then scaling the choice variables by $(1 - \frac{c_t}{E_t})E_t$ and defining

$$\bar{c}_t = \frac{c_t}{E_t},$$

the real return on equity is express as

$$R^e_{t+1} \equiv \frac{1}{(1-\bar{c})E_t}\left(r_{t+1}\tilde{b}_{t+1} + (r^{ior}_{t+1} - tax_t)\tilde{m}_{t+1} - r^d_{t+1}\tilde{d}_{t+1} + \chi_{t+1}(\tilde{m}_{t+1}, \tilde{d}_{t+1}, \omega|\theta_t)\right).$$

For a foreign bank, the tax is zero, and the liquidity cost function denoted by χ is specific to each type of bank and depends on both the idiosyncratic shock of each bank and the resulting market tightness denoted by θ_t.

By substitution

$$v_t E_t^{1-\eta} = \max{}_{\tilde{b}_t, \tilde{m}_t, \tilde{d}_t \leq 0}\left[\frac{(\bar{c}_t E_t)^{1-\eta}}{1-\eta} + \beta((1-\bar{c}_t)E_t)^{1-\eta} \mathbb{E}_{\omega}[v_{t+1}(R^e_{t+1})^{1-\eta}]\right],$$

subject to

$$\frac{\tilde{b}_{t+1} + \tilde{m}_{t+1} - \tilde{d}_{t+1}}{(1-\bar{c})E_t} = 1, \tag{34}$$

and

$$\frac{\tilde{d}_{t+1}}{(1-\bar{c})E_t} \leq \kappa. \tag{35}$$

Define

$$\Omega_t \equiv \max_{\{\tilde{b}_{t+1}, \tilde{m}_{t+1}, \tilde{d}_{t+1}, c_t\}} \mathbb{E}_{\omega}\left\{{R^e_{t+1}}^{1-\eta}\right\}^{\frac{1}{1-\eta}}$$

It follows that

$$v_t = \frac{1 + (\beta(1-\eta)\Omega_t^{1-\eta})^{1-\eta}}{1-\eta},$$

and from the first-order conditions of the maximization with respect to $\bar{c}_t$

$$\bar{c}_t = \frac{1}{1 + (\beta v_{t+1}(1-\eta)\Omega^{1-\eta})^{1/\eta}}$$

For proofs and derivation, see Bianchi and Bigio (2014).

In what follows, I assume the log-linear utility as $\eta \to 1$. Then the problem can be characterized with

1. A single bellman equation for each type of bank

$$\Omega_t^i = \max_{\bar{b}_t^i, \bar{m}_t^i, \bar{d}_t^i \leq 0} \exp\left\{ \mathop{\mathbb{E}}_{\omega^i, \omega^j} [ln\big(R_{t+1}^{ei}(\bar{b}_{t+1}^i, \bar{m}_{t+1}^i, \bar{d}_{t+1}^i)\big)]\right\},$$

   subject to the scaled balance sheet constraint, and scaled capita constraint and non-negativity constraint as in (34), (35), and (4) for domestic banks with $[\bar{b}_t^i, \bar{m}_t^i, \bar{d}_t^i] = (1-\bar{c}_t^i)E_t^i[\tilde{b}_t^i, \tilde{m}_t^i, \tilde{d}_t^i]$, and

$$\Omega_t^j = \max_{\bar{b}_{t+1}^j, \bar{m}_{t+1}^j, \bar{d}_{t+1}^j \leq 0} \exp\left\{ \mathop{\mathbb{E}}_{\omega^i, \omega^j} [ln\big(R_{t+1}^{ej}(\bar{b}_{t+1}^j, \bar{m}_{t+1}^j, \bar{d}_{t+1}^j)\big)]\right\},$$

   subject to the relevant (34), (35), and (**??**) for foreign banks with $[\bar{b}_t^j, \bar{m}_t^j, \bar{d}_t^j] = (1-\bar{c}_t^j)E_t^j[\tilde{b}_t^j, \tilde{m}_t^j, \tilde{d}_t^j]$,

2. the value functions

$$V_t^i(E_t^i) = v_t^i ln(E_t^i),$$

   and

$$V_t^j(E_t^j) = v_t^j ln(E_t^j)$$

   where

$$lim_{\eta \to 1}(1-\eta)v_t = 1/(1-\beta).$$

3. So that the optimal bank equity dividend ratios are

$$\frac{c_t^i}{E_t^i} = 1 - \beta$$

and

$$\frac{c_t^j}{E_t^j} = 1 - \beta.$$

Because the policy functions are linear in their equity, two banks of the same type with different equity levels are a scaled version of a bank with one equity unit. In other words, only aggregate equity of each type is a state variable for the banking side of the model with no account for the distribution of equity between individual banks within a sector; the steady-state results measure the portfolio shares of each type of representative bank. In this sense, economic aggregates, such as the steady-state solution, the distribution of reserves, and interbank lending activity of each type of bank are scaled by the sector's size. For example, if we had $\$5$ billion worth of equity for domestically charted banks and $\$1$ billion associated with U.S. foreign branches and agencies, then the aggregates for each sector are multiplied by the sectors' respective relative size of 5:1.

In the BB model, all banks are facing the same portfolio problem and same interbank market conditions. Alternatively, in the DF model, the cost function and the withdrawal shock differ across the two sectors. Because a bank's decision is based on the expected liquidity cost, it depends on the given outside rates available to each type of bank and the market tightness. The latter is endogenous to the aggregate interbank market conditions that depend on the optimal choice of each type of bank, and therefore for may change the outcome of the federal funds rate.

For example, looking at the first-order conditions for reserves,

$$\frac{\partial \Omega}{\partial m} : R^b - R^m = \mathop{\mathbb{E}}_{\omega}\left(\frac{\partial \chi_t(\cdot)}{\partial m}\right) + \frac{COV_{\omega}\left((R^e)^{-1}, \frac{\partial \chi_t(\cdot)}{\partial m}\right)}{\mathbb{E}_{\omega}\left(R_t^e\right)^{-1}},$$

we see that banks choose to increase reserves to the point that the marginal cost of reserves (which is the opportunity cost of not lending the funds for the illiquid loan return of Rb) equals the liquidity benefit/cost associated with lending/borrowing reserves in the federal funds market. The first term is simply the marginal cost or benefit given by the market conditions, and the second term is the risk premium associated with the withdrawal shock. If foreign banks choose large excess reserves (either because of the higher risk of withdrawals or the differential of overnight rates), it will lower domestic banks' deficit cost, increasing their opportunity cost of reserves.

4.2 Evolution of Equity and Equity Growth for Stationary Equilibrium

To establish the law of motion for the aggregate equity of each type of bank, we integrate across banks the definition of its equity and iterate one period forward to arrive at

$$E^D_{t+1} = \tilde{M}^D_{t+1}(r^{ior}_{t+1} - tax_{t+1}) + \tilde{B}^D_{t+1} r_{t+1} - \tilde{D}^D_{t+1} r^d_{t+1} - DW^D_{t+1}(r^{dw}_{t+1} - r^{ior}_{t+1} - tax_{t+1}) - X^D_{t+1}(r^{ff}_{t+1} - r^{ior}_{t+1} - tax_{t+1}) \tag{36}$$

and

$$E^F_{t+1} = \tilde{M}^F_{t+1} r^{ior}_{t+1} + \tilde{B}^F_{t+1} r_{t+1} - \tilde{D}^F_{t+1} r^d_{t+1} - DW^F_{t+1}(r^{dw}_{t+1} - r^{ior}_{t+1}) - X^F_{t+1}(r^{ff}_{t+1} - r^{ior}_{t+1}). \tag{37}$$

Because the model is scale-invariant, the steady-state solution must only keep track of the evolution of average equity of each of the two banking sides. Define the average equity of each sector as $\bar{E}^i \equiv \frac{1}{share}\int_0^{share} E^i di$ and $\bar{E}^j \equiv \frac{1}{1-share}\int_{share}^1 E^j dj$. Using the equation for the domestic surplus, equation 5, for every E^i_t, there is a common $\omega^*_i = \frac{\rho - \bar{m}^i_{t+1}/\bar{d}^i_{t+1}}{r^d_{t+1}/(r^{ior}_{t+1} - tax_{t+1}) - \rho}$, implying a mass of reserves deficit in the domestic sector given by

$$I^- = \mathbb{E}_{\omega^i}[s(x(\omega^i))|\omega^i < \omega^*_i] D\left[\frac{\rho - \bar{m}^i_{t+1}/\bar{d}^i_{t+1}}{r^d_{t+1}/(r^{ior}_{t+1} - tax_{t+1}) - \rho}\right]\bar{E}^i_t \tag{38}$$

and a mass of the domestic surplus of reserves following

$$I^+ = \mathbb{E}_{\omega^i}[s(x(\omega^i))|\omega^i > \omega^*_i]\left(1 - D\left[\frac{\rho - \bar{m}^i_{t+1}/\bar{d}^i_{t+1}}{r^d_{t+1}/(r^{ior}_{t+1} - tax_{t+1}) - \rho}\right]\right)\bar{E}^i_t. \tag{39}$$

Similarly, using the equation for foreign surplus in equation 6, for every E^j_t, there is a common $\omega^*_j = \frac{-\bar{m}^j_{t+1}/\bar{d}^j_{t+1}}{r^d_{t+1}/r^{ior}_{t+1}}$, so that the mass of foreign deficit and surplus of reserves follows

$$J^{-} = \underset{\omega^j}{\mathbb{E}}[s(x(\omega^j))|\omega^j < \omega_j^*]F\left[\frac{-\bar{m}_{t+1}^j/\bar{d}_{t+1}^j}{r_{t+1}^d/r_{t+1}^{ior}}\right]\bar{E}_t^j \tag{40}$$

and a mass of the foreign surplus of reserves following

$$J^{+} = \underset{\omega^j}{\mathbb{E}}[s(x(\omega^j))|\omega^j > \omega_j^*]\left(1 - F\left[\frac{-\bar{m}_{t+1}^j/\bar{d}_{t+1}^j}{r_{t+1}^d/r_{t+1}^{ior}}\right]\right)\bar{E}_t^j. \tag{41}$$

To arrive at a growth rate, rewrite the law of motion of aggregate equity in equations 36 and 37 given the definitions of real returns for each type of bank, and substitute the balancing-stage overnight funds as in equations 21 and 25, given the scale-invariant properties of banks' mass of reserves such that,

$$\begin{aligned} E_{t+1}^D =& \big(\bar{m}_{t+1}^i(r_{t+1}^{ior} - tax_{t+1}) + \bar{b}_{t+1}^i r_{t+1} - \bar{d}_{t+1}^i r_{t+1}^d - I^{-}(1-\gamma^{-})(r_{t+1}^{dw} - r_{t+1}^{ior} + tax_{t+1}) \\ & - [I^{-}\gamma^{-} - I^{+}\gamma^{+}](r_{t+1}^{ff} - r_{t+1}^{ior} + tax_{t+1})\big)E_t^D(1-\bar{c}^i) \end{aligned} \tag{42}$$

and

$$\begin{aligned} E_{t+1}^F =& \big(\bar{m}_{t+1}^j r_{t+1}^{ior} + \bar{b}_{t+1}^j r_{t+1} - \bar{d}_{t+1}^j r_{t+1}^d - J^{-}(1-\gamma^{-})(r_{t+1}^{dw} - r_{t+1}^{ior}) \\ & - [J^{-}\gamma^{-} - J^{+}\gamma^{+}](r_{t+1}^{ff} - r_{t+1}^{ior})\big)E_t^F(1-\bar{c}^j). \end{aligned} \tag{43}$$

It follows that the equity growth is equal to

$$\begin{aligned} E_g^D \equiv& \beta\big(\bar{m}_{t+1}^i(r_{t+1}^{ior} - tax_{t+1}) + \bar{b}_{t+1}^i r_{t+1} - \bar{d}_{t+1}^i r_{t+1}^d \\ & - I^{-}(1-\gamma^{-})(r_{t+1}^{dw} - r_{t+1}^{ior} + tax_{t+1}) - [I^{-}\gamma^{-} - I^{+}\gamma^{+}](r_{t+1}^{ff} - r_{t+1}^{ior} + tax_{t+1})\big) \end{aligned}$$

and

$$E_g^F \equiv \beta\big(\bar{m}_{t+1}^j r_{t+1}^{ior} + \bar{b}_{t+1}^j r_{t+1} - \bar{d}_{t+1}^j r_{t+1}^d - J^{-}(1-\gamma^{-})(r_{t+1}^{dw} - r_{t+1}^{ior}) - [J^{-}\gamma^{-} - J^{+}\gamma^{+}](r_{t+1}^{ff} - r_{t+1}^{ior})\big).$$

In a steady-state stationary equilibrium, the equity growth rate is zero, so that

$$E_g^D = E_g^F = 1.$$

5. Results

5.1 Data and Parameter Calibration

Calibration of bank-specific parameters uses the Federal Financial Institutions Examination Council (FFIEC) quarterly filings. FFIEC 031, Reports of Condition and Income (also known as the Call Reports) for domestically chartered banks, and FFIEC 002, the Report of Assets and Liabilities for branches and agencies of foreign banking organizations. The federal funds market operates daily, and ideally, one would calibrate the specific withdrawal of banks using daily data on the operations of the federal funds market. Such data exists for domestically chartered banks but is not available for foreign branches and agencies before 2016. The lack of earlier data on foreign banks is a problem because the data post the introduction of IORB does not reveal the liquidity constraints of financial institutions and henceforth is not applicable for our purpose.

Instead, I impute daily volatility for foreign banks using the ratio of quarterly volatility across foreign and domestic banks, measured by the cross-sectional deviation from the mean of each sector, and then multiplied by the daily volatility of domestic banks. Afonso and Lagos (2015) measure the volatility to equal 0.05 based on the daily volumes of federal funds traded as reported in FR 2420 by a sample of 134 banks.[21]

Table 2 reports the summary statistics of the two sectors' total deposits, transaction accounts, non-transaction accounts, and demand deposits, as measured between 2005 Q1 to 2011 Q4. We want to measure the banks' liquidity needs, ide-

[21] These banks are all domestically charted since, as mentioned, foreign branches and agencies were not required to fill this form during this study period. The volumes are reported daily by financial institutions in FR 2420 form but are confidential. The available data on the daily volume is aggregated over all banks. Beginning June 2016, as part of Dodd-Frank Act recommendations, the Fed required that FBOs with total consolidated assets of $\$50$ billion or more establish a U.S. intermediate holding company (IHCs). These companies must file specific reports such as FR Y-9 post the new legislation so that the aggregate volume is now available separately for domestic and foreign banks.

ally measured by total deposits. However, since foreign branches and agencies do not file the same report as domestically chartered banks, total deposits are not comparable between the two sectors,[22] and thus, the observed difference between the two columns could result from the differences in the reporting form. Demand deposits are not best suited to measure actual liquidity since these are deposits for which most transactional activities occur. However, it lends a good proxy for the difference in the volatility ratios of total deposits because they are defined identically between the two reporting forms.

The cross-sectional deviation of demand deposits is 0.210 for domestically chartered banks and 0.502 for foreign branches and agencies or 2.3 times larger for foreign banks, as shown in Table 2. Given the empirical evidence that domestic withdrawal volatility is $\sigma^D = 0.05$ (Afonso and Lagos, 2015) and the proportion in fluctuations between the two sectors, we estimate the deviation of the withdrawal distribution for foreign banks to be $\sigma^F = 0.115$.

Figure 5 shows how a ceteris paribus increase in the volatility will increase reserves in response to a policy of interest on reserve balances. We see that with the current calibration of the volatilities of the two sectors, uninsured banks slope is steeper. The steep slope corresponds with a greater increase in reserves to assets following a change in the IORB rate.

Other parameters of the federal funds market are market tightness and the bargaining parameter. The market tightness reflects the rate at which orders are matched, and it is equal to $\lambda = 2.1$, as documented by Bianchi and Bigio (2014). They calibrate this by the same empirical evidence presented in Afonso and Lagos (2015) and set it to match the Fed's fraction of discount window loans as a fraction of the total reserves. The bargaining power ϕ_t is endogenous and depends on whether lending orders are greater than borrowing orders or vice versa. The benchmark $\bar{\phi} = 0.5$ is set such that when lending and borrowing orders are equal

[22]Domestically chartered banks report total deposits, while foreign branches and agencies report total deposits and credit balances. Domestically chartered banks report the total transaction accounts, while foreign branches and agencies report the total transactions accounts and credit balances. Similarly, domestically chartered banks report the total transaction accounts, while foreign branches and agencies report the total transaction accounts and credit balances.

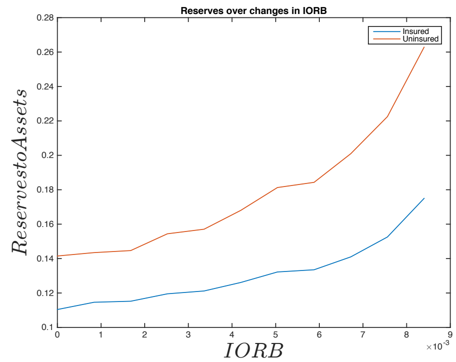


Figure 5: Volatility of Deposit Withdrawals and IORB

The reserve to asset ratio corresponding with different IORB rates. The red line for a higher deposit withdrawal volatility and the blue corresponds to a lower volatility

$\phi_t = \bar{\phi}$, and the federal funds rate is equal to the midpoint of the banks' second-best rates.[23]

Table 2: Deposits Volatility Summary Statistics

	Domestically chartered banks			Foreign branches and agencies		
Variable	Mean[1]	Std. Dev.[2]	N[3]	Mean	Std. Dev.	N
Total Deposits*	-0.002	0.032	170912	-0.020	0.437	4424
Transaction Accounts**	-0.006	0.184	170912	-0.034	0.717	4396
Demand Deposits	-0.008	0.210	171164	-0.049	0.502	4256
Non-Transaction Accounts	-0.003	0.053	171164	-0.054	0.643	4564

[1] The mean is the average of the cross-sectional means of the deviation from deposit growth at each period.
[2] The Std.Dev is the average of the standard deviation of the cross-sectional deviation from deposit growth at each period.
[3] N changes across variables since I use the convention of dropping individual banks that do not report that variable for any period.
* Total deposits of foreign branches and agencies are defined as total deposits and credit balances, and therefore may not be comparable to that of a domestic bank.
** Total transaction accounts of a foreign branch are defined as total transaction accounts and credit balances, and therefore may not be comparable to that of a domestic bank.

Table 3 four dates compare two policy regimes implemented during two different periods: prior and post the introduction of IORB and the change in the FDIC assessment. Steady-state values are reported in the next section. The first policy's reference dates are 2008 Q3 and 2008 Q4, that is, before and after introducing in-

[23]Appendix D describes the function for ϕ in more detail.

terest on reserves. Furthermore, the reference dates for the FDIC policy are 2011 Q2 and 2011 Q3, respectively. The FDIC policy was implemented in April of 2011, but the data suggest that banks did not adjust to the policy before the third reporting quarter of this year.

The first part of the table summarizes the parameters that stay constant throughout the different policy regimes and are consistent with those calibrated by Bianchi and Bigio (2014). Other than the federal funds market parameters already discussed, these include bank preferences parameters denoted by $\{\beta, \eta\}$, and regulatory parameters $\{\kappa, \rho\}$. The discount factor $\beta = 0.981$ is calibrated to match a dividend capital interest ratio of 8% consistent with the literature, and the risk aversion $\eta = 1$, implies a constant dividend-equity ratio. Both parameters are assumed to be the same across the two sectors. The regulatory parameters $\kappa = 10$ and $\rho = 0.1$ are consistent with a leverage ratio and reserve requirements of 90% capital and 10% reserves.

The relative size of domestic, commercial banks to U.S. branches of foreign banks is denoted by $share$ and measured to be relatively constant at 85% across the study period. This parameter is necessary to adequately estimate the probabilities of meeting different borrowers or lenders in the interbank market. The extension to the model, including GSEs, calibrates their relative size to banks at around 40% during the relevant period and is denoted by a. Overnight loans by GSEs in the federal funds market dominate the market during the Fed's large-scale asset purchases (Craig and Millington, 2017). However, an exact number for the number of loans is not available. The total cash assets of GSEs are estimated to be $\$250$ billion (Afonso et al., 2019), so that overnight loan orders are estimated to equal 5% of GSEs total equity based on the number of aggregate cash assets to total assets held by GSEs.

The second part of Table 3 summarizes the parameters that change between the four periods and includes regulatory parameters $\{i^{ior}, i^{dw}, tax\}$, which are the interest on reserves, the discount window rate, and the FDIC tax assessment respectively. The IORB is calibrated to that from FRED, Federal Reserve Bank of St. Louis. The discount window rate is calibrated to include an additional 44 monthly

Table 3: Calibration

	Value	Source
Discount Factor	$\beta = 0.981$	Dividend ratio of 8%: $\beta - 1 = 1 - (1+0.08)^{1/4}$
Risk Aversion	$\eta = 1$	Constant dividend-equity ratio.[1]
Bargaining Parameter	$\bar{\phi} = 0.5$	Equal bargaining power when lending orders equal borrowing orders
Matching Friction	$\lambda = 2.1$	DW to reserves of 2%[2]
Capital Requirement	$\kappa = 10$	Regulatory requirement of 90% capital ratio
Reserves Ratio	$\rho = 0.1$	Regulatory requirement of 10% required reserves ratio
Insured-deposit Volatility	$\sigma^D = 5\%$	FR 2420, daily data on interbank activity[3]
Uninsured-deposit Volatility	$\sigma^F = 11.5\%$	Call reports, quarterly data of financial standing [4]
Domestic to Foreign Asset Share [5]	$share = 0.85$	Board of Governors H.8 and Z.1
GSEs' Asset Share[6]	$a = 0.4$	Board of Governors L.125
GSEs' Interbank Lending[6]	$G = 5\%$	Board of Governors L.125

	2008 Q3 (No IORB)	2008 Q4 (IORB)	2011 Q2 (No FDIC)	2011 Q3 (FDIC)
Discount Window Rate	2.25%	1.30%	0.75%	0.75%
Interest on Reserve Balances	$--$	0.80%	0.25%	0.25%
FDIC Assessment on Reserves[7]	$--$	$--$	$--$	0.15%

[1] Implies that substitution and income effects cancel out and that the amount of dividends is only a function of equity.
[2] The matching friction was calibrated in Bianchi and Bigio (2014) to match empirical evidence from the federal funds market. It functions as the Poisson process rate of arrivals governing the fraction of interbank market orders traded. They set this to match the observation of discount window loans equal to 2% of total reserves in 2006.
[3] Measured as the mean of the cross-sectional standard deviation of non-transactional accounts.
[4] Foreign banks are subject to larger withdrawals in the event of a bank run because they are not eligible for deposit insurance, meaning that although the average distribution of inflows/outflows of funds to the bank is the same across sectors, the risk of withdrawals is always higher.
[5] This share changes during the three regimes - with out loss of generality the ratio is the average during the study period.
[6] The share of GSEs in the federal funds market is also calibrated as an average for the whole time period.
[7] The actual rate is based on bank's size, where larger banks have a higher assessment.

basis points than the reported rate estimated by Armantier et al. (2015). They document the stigma premium from the rate in the Term Auction Facility. The Term Auction Facility is an additional lending facility for banks to avoid borrowing from the discount window because borrowing from the Fed renders a bank unstable and risky, given it is a means of last resort.[24]

The FDIC assessment base rate is calibrated to approximate the average rate paid by the average bank. In practice, CAMELS ratings are used to determine the

[24] Armantier et al. (2015) observe that banks will choose to pay on average 44 basis points more on a 30-day loan in the federal funds market to avoid the stigma associated with borrowing from the central bank. However, they note that only the rate paid is observable. Therefore, the estimated stigma is a lower bound on what banks are willing to pay in the TAF facility. The estimated lower bound of this premium was as high as 146 basis points during the Lehman Brothers bankruptcy and could potentially be much more significant.

risk category for a given bank. Riskier or larger banks pay more, meaning that the policy may have or may not have changed the individual assessment banks pay. However, it shifted costs from deposits to holding excess reserves and, more importantly, changed funding strategies. In what follows, we explore this effect on the average bank.[25] The actual tax may range from 2.5 to 45 basis points depending on the bank's risk category. For example, the assessment rate is between 9-24 basis points for a low to medium risk rating (Whalen, 2011). This exercise follows Kotok (2011) approximation that the new assessment aggregate response was equivalent to a tightening of 15 basis points, or a $tax = 0.15\%$, which is a reasonable assumption since it lies in the mid-range.

5.2 Steady-State Results

The steady-state tables below provide four quantitive exercises. First, we look at the fit of the model against the observed aggregate interbank data and each sector's aggregate reserve ratios during four reference periods. Table 4 compares Q3 of 2008 to Q4 of the same year, while Table 5 reports the subsequent Q2 of 2011 and Q3 of 2011.[26] We compare these periods to embed two critical changes in the model: the period with interest on excess reserves (introduced to bound historically low-interest rates away from zero) and the period for which the FDIC assessment base changed. The FDIC policy essentially created a difference in the outside lending option facing domestically chartered banks to foreign branches and agencies. The second quantitative exercise examines some of the key differences of the single representative bank model and the Baseline model (with two banking sectors) to the Extended version, including GSEs. Table 6 shows that the implications of monetary policy of interest on excess reserves are different across the three models. In Tables 7 and 8, we use counterfactual parameters to compare the implication of the two features distinguishing domestic from foreign banks and

[25] Although a detailed model of banks with different rates based on their risk categories is of great interest in assessing if this policy results in financial stability, it is out of this paper's scope.

[26] The exception is that in the two tables the yearly discount window rate is 44 monthly basis points higher than the reported discount window rate, as Armantier et al. (2015) estimated due to stigma that may signal to a low-quality borrower.

estimate which of the two channels embedded in the model is prominent in the observed outcome of the interbank market. Following this are other counterfactuals to estimates the long-run effects of a change in additional policy tools on the market and the bank's optimal choice. These tools include the overnight reserve repurchase rate for GSEs in Table 9, the discount window rate in Table 10, the FDIC assessment rate in Table 11, and the withdrawal shock of banks in Table 12.

Before 2008 the federal funds market operated under a scarcity of funds. After that, large-scale asset purchases by the Fed increased the abundance of available overnight funds resulting in a satiated market. In addition, because GSEs could not receive interest on excess reserves, they became the absolute lenders of overnight funds during this period. We document that bank lending in the interbank market fell from over 50% during 2006 to less than 20% by 2012. One way to calibrate this situation is by adding an exogenous level of lending orders at each iteration. Meaning that no matter the banking side choice of reserves, following the withdrawal shock, total lending orders include those by banks and an additional exogenous amount by GSEs. GSEs' lending is assumed to face a second-best lending rate of ON RRP rate, equal to zero until September 2013. Below, the Baseline model refers to the two-sector model with no GSEs, and the Extended model, to that including GSEs facing an ON RRP rate equal to zero, while the effects of a change in the ON RRP rate are reserved for Table 9.

5.2.1 Steady-state across the two policy changes:

The three columns in Tables 4 and 5 correspond to the data, Baseline, and Extended models. The first two rows consist of lending-stage portfolio choices of each representative bank with reserves to assets equaling the ratio of liquid assets to total assets as reserves are the only liquid asset in the model. On the other hand, the reserves to asset ratio for the data column is aggregated from the individual financial Call Reports. Similarly, the reserve ratio in the model is a measure of bank liquidity, while the reserve to deposit ratio is from the Call Reports. This comparison can be misleading because banks hold various liquid assets, but it gives insight into the model's bearing.[27]

The second part of the following two tables reports the shares of interbank market activity with data on interbank borrowing and lending available from the Federal Reserves Bank of New York. Only banks generally generate interbank borrowing since they need them to clear transactions. The total borrowing share is the share of the two sectors. In contrast, interbank lending is available from different financial institutions, where historically GSEs have been the major lenders of funds. The share of bank lending is the share of each sector as a fraction of only bank lending and is comparable to the interbank lending in the Baseline model. Total lending share is the share of each sector from the entire pool of lending observed in the market and is comparable to the interbank lending share in the Extended model, which includes GSEs. The last part of these tables reports the data and estimations of discount window loans to reserves and the interbank market rate (the effective federal funds rate) reported by the Federal Reserve Board of Governors and retrieved by the FRED website. The last row provides the estimated overall market tightness.[28]

Table 4 is partitioned into two parts, first comparing the parameter specification of the third quarter of 2008 and below comparing the effect of IORB policy in the last quarter of 2008. We generally find somewhat better estimations with the Extended model in the third column across both periods. However, before the policy change, the models' predictions of domestic reserves were too high, while foreign reserves were too low. As noted, because reserves are the only liquid asset in the model, this is expected. Similar conclusions apply to the ratios of the reserve to deposits. Notably, GSEs' lending reduces the ratios substantially as the market tightness is lower with their presence. Therefore, we see the liquidity premium is also lower with the Extended model, but not low enough to match the empirical

[27]The appendix reports the data of liquid assets holdings, including securities of each banking sector as a function of total assets and total deposits. Although this data is more closely estimated with the Extended model, I do not use these for the comparison as the items on each of the two sectors' balance sheets are slightly different.

[28]Market tightness equals $100 \times \frac{\theta}{(1+\theta)}$ and reports the probability of matching a lending order. Thus the higher the probability, the higher the scarcity of overnight funds, the larger the bargaining power of lenders, and vice versa.

Table 4: Steady-state Calibrated to 2008 Q3 and 2008 Q4

	Data 2008 Q3		Baseline*		Extended*	
	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign
Reserves to Assets	1.6 %	0.5%	9.4%	1.2%	5.3%	0.1%
Reserve Ratio	2.4%	0.9%	10.4%	1.3%	5.8%	0.2%
Total Borrowing Share	55.7%	44.3%	70.0%	30.0%	85.4%	14.6%
Bank Lending Share	42.4%	57.6%	68.1%	31.9%	–	–
Total Lending Share	15.8%	21.5%	–	–	1.7%	3.0%
Discount Window Share	0.9%		0.2%	3.7%	1.0%	33.6%
Interbank Rate	1.94%		2.85%		2.34%	
Market Tightness	–		44.1%		22.5%	
	Data 2008 Q4		**Baseline***		**Extended***	
	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign
Reserves to Assets	4.4%	6.5%	11.6%	6.6%	11.2%	5.6%
Reserve Ratio	6.6%	10.3%	12.7%	7.3%	12.3%	6.2%
Total Borrowing Share	67.5%	32.5%	70.7%	29.3%	68.9%	31.1%
Bank Lending Share	70.5%	29.5%	68.6%	31.4%	–	–
Total Lending Share	23.2%	9.7%	–	–	0%	0%
Discount Window Share	0.5%		0.1%	0.3%	0.4%	0.4%
Interbank Rate	0.51%		2.57%		1.91%	
Market Tightness	–		17.5%		21.0%	

* The table compares two models to the data: The Baseline model consisting of foreign and domestic banks and the Extended model of two bank sectors and GSEs

data on the federal funds rate (moving from 2.34% to 1.91% compared to the empirical average of 1.91% going down to 0.97% across the two periods).

The estimations would be better with higher liquidity (calibrating more available lending from GSEs) or a lower stigma associated with the discount window rate. Both these calibrations are not perfect empirical measures. The amount of aggregate cash to assets and the ratio of GSEs assets to bank assets are used to estimate GSEs lending share in the absence of a better measure. Likewise, the stigma may depend on the scarcity of funds (meaning it might be higher at times of aggregate financial distress). Appendix A includes Tables 13 and 14, with an alternative specification of these tables having no additional' stigma' premium and are consistent with the general results, albeit the optimal reserves of both sectors and the federal funds rate are low in this case.

Above all, the models capture well the change in reserve ratios in response to the policy change. As seen in the second part of the table, the inclusion of IORB implies the ratios of domestic banks doubled while those of foreign banks increased from 0.5% to 6.5%. Although domestic reserves are still too high in the model, the ratios double, explicitly from 5.3% to 11.2% in the Extended model. The Baseline model's predictions are conversely less consistent with the data moving from 9.4% to 11.6%. However, foreign reserves ratios in both models are within the magnitude and rate of change of the empirical evidence.

In addition, interbank shares estimated by the model are comparable to the data for the period following the IORB policy. Domestic bank lending share estimate is around 69% compared to 71% in the data, and the borrowing share is around 70.0% compared to 74%. The total lending share in the third column of the Extended model is equal to zero while we observe a decline in foreign bank lending following policy but an increase in the share of domestic lending. Still, the data on interbank lending is limited because it only reports interbank loans of Federal Home Loan banks rather than all lending by GSEs- so again, this will not be a perfect match. Historically GSEs such as Fannie Mae and Freddie Mac reports federal funds jointly with repo transactions and therefore are excluded from the available data during this period. In practice, we expect the lending of GSEs to take up a more significant share of total lending than that reported in the table.

Table 5 repeats the comparison but coincides with the two reference periods before and preceding the FDIC policy change. The change in the deposit insurance assessment base increased the balance sheet cost associated with domestic reserves. Therefore, it essentially differentiated the outside lending rate across domestic and foreign banks. The data shows very high reserve ratios for the foreign sector that are nowhere close to what is estimated by the two models, as seen in columns two and three of this table. Even so, we find the FDIC policy change in the model estimates foreign banks will slightly increase or not change reserve ratios, and domestic banks will slightly reduce them. We will see in the preceding counterfactual steady-state that the tax on domestic reserves can replicate the ratios of reserves held by banks, but these necessitate a higher interest on reserves and a

Table 5: Steady-state Calibrated to 2011Q2 and 2011 Q3

	Data 2011 Q2		Baseline*		Extended*	
	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign
Reserves to Assets	5.9 %	31.5%	11.4%	6.2%	10.3%	3.4%
Reserve Ratio	8.2%	59.7.8%	12.5%	6.8%	11.3%	3.8%
Total Borrowing Share	38.5%	61.5%	70.7%	29.3%	70.4%	29.6%
Bank Lending Share	65.5%	34.5%	68.6%	31.4%	–	–
Total Lending Share	10.6%	5.6%	–	–	8.5%	4.0%
Discount Window Share	0.97%		0.1%	0.3%	0.1%	0.9%
Interbank Rate	0.09%		2.00%		1.82%	
Market Tightness	–		19.1%		8.2%	
	Data 2011 Q3		Baseline*		Extended*	
	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign
Reserves to Assets	5.1%	34.6%	10.9%	6.6%	9.8%	3.4%
Reserve Ratio	7.0%	76.7%	12.0%	7.2%	10.8%	3.7%
Total Borrowing Share	54.6%	45.3%	75.6%	24.4%	73.6%	26.4%
Bank Lending Share	64.4%	35.6%	65.4%	34.6%	–	–
Total Lending Share	11.6%	6.4%	–	–	7.5%	4.0%
Discount Window Share	0.72%		0.1%	0.3%	0.2%	1.0%
Interbank Rate	0.08%		1.95%		1.81%	
Market Tightness	–		22.0%		9.3%	

* The table compares two models to the data: The Baseline model consisting of foreign and domestic banks and the Extended model of two bank sectors and GSEs

higher assessment cost. Moreover, the reduction in reserve ratios is coupled with a slightly lower interbank rate; however, still too high.

The comparison of the two policies concludes that the model predictions are more robust as reserves become the dominant liquid assets on the bank portfolio. In addition, we find that the calibrated share of each sector's size and the trade-offs each face allows for reasonable estimations of the interbank market activity before the FDIC policy change but not after. The effect of massive quantitive easing during these two periods may contribute to the discrepancies since in the absence of such liquidity, the interbank market is too tight, the interbank rate is too high, and foreign banks will not gain from borrowing overnight funds at a rate below the IORB.

5.2.2 Steady-states across three models:

Although estimations fall short of the empirical evidence, the previous tables showed significant differences in the response to policy change across models. These differences are found to have aggregate policy implications. In the following steady-state table, a comparison of models shows the multi-sectoral economy implications for policy. The two columns in Table 6 correspond to before and after a change in monetary policy. There is no interest on reserves in the first column, while in the second column, the interest is 0.5%, and the FDIC tax on domestic reserves is 0.15%. Each column is divided into three, comparing outcomes of the One Bank model, the Baseline, and the Extended model. Since the representative bank in the One Bank model is assumed to be a domestic bank, outcomes are compared with the domestic sector of the other two models in the first part of the table. In the second part, a single representative foreign bank is compared to the two other models.

In the One Bank and the Baseline model, reserves ratios increase while market tightness declines, and the interbank rate is higher in response to the policy change. In comparison, the Extended model estimates of the percentage change in reserve ratios are more considerable and coupled with a higher market tightness. The reason is that a rate differential across different lenders will increase the demand for overnight borrowing. The decline in the interbank rate, in this case, is due to the more considerable increase in reserves. Hence, a rate differential across interbank lenders that increases banks' demand for overnight borrowing mitigates the policy effect of IORB. In effect, another question of interest postulates whether an increase in the observed higher reserves held by foreign banks is due to a more considerable risk of withdrawal rather than the interest rate differential. Tables 7 and 8 explore this question using a set of counterfactual parameters for the volatility risk and FDIC tax.

Table 6: Steady-state Across Three Models

	No IORB			IORB= 0.5%, tax=0.15%		
Domestic Bank	One Bank	Baseline	Extended	One Bank	Baseline	Extended
Reserve to Assets	9.4%	9.5%	5.8%	10.0%	10.1%	9.4%
%Δ				(6.4%)	(6.3%)	(62.1%)
Market Tightness	45.5%	42.9%	20.6%	37.2%	28.9%	22.6%
%Δ				(−18.2%)	(−32.6%)	(9.7%)
Interbank Rate	1.17%	1.10%	0.92%	1.24%	1.23%	0.87%
Δ				(0.07)	(0.13)	(-0.05)
	No IORB			IORB= 0.5%, tax=0.15%		
Foreign Banks	One Bank	Baseline	Extended	One Bank	Baseline	Extended
Reserve to Assets	1.2%	1.8%	0.4%	5.4%	6.2%	3.7%
%Δ				(350%)	(244%)	(825%)
Market Tightness	39.4%	42.9%	20.6%	20.8%	28.9%	22.6%
%Δ				(−47.2%)	(−32.6%)	(9.7%)
Interbank Rate	1.03%	1.10%	0.92%	1.3%	1.23%	0.87%
Δ				(0.27)	(0.13)	(-0.05)

* The table compares implications of monetary policy of IORB across three types of models: the One Bank model, the Baseline model consisting of foreign and domestic, and the Extended model consisting of two bank sectors and GSEs.

5.2.3 The two channels that affect bank tradeoffs:

In Tables 7 and 8, the three columns compare three cases: in column one, both sectors have the same risk with interest on reserve balances but no tax; in column two, balance-sheet costs are the same ($tax = 0$), but the risk of a foreign bank's withdrawal is higher than that of a domestic bank; and in column three domestic bank's balance-sheet costs are higher with the same risk across sectors. Table 7 presents the Baseline model results and Table 8 that of the Extended model with GSEs. The discount window rate is set to 4%, the interest on reserves to 0.5%, and if a tax is present, it is set to equal 0.15%, consistent with the average rate in the data. Similarly, the withdrawal volatility parameter σ is equal to 0.05 for both banks, or it is 2.3 times larger for the foreign sector as is calibrated by the data present in Table 2.

We find that even in the absence of a higher risk of withdrawals, an FDIC tax for domestic banks (or higher balance-sheet cost) implies foreign banks will sub-

Table 7: Calibrating a Different Risk of Withdrawals and an FDIC Rate

	$\sigma^d = \sigma^f = 0.05,\ \ tax = 0$		$\sigma^d = 0.05, \sigma^f = 0.113,\ \ tax = 0$		$\sigma^d = \sigma^f = 0.05,\ tax = 0.15\%$	
	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign
Reserve to Assets	11.2%	2.4%	11.2%	5.8%	10.6%	2.8%
Reserve Ratio	12.3%	2.7%	12.3%	6.4%	11.6%	3.1%
Interbank Borrow Share	84.0%	16.0%	72.1%	27.9%	88.6%	11.4%
Interbank Lending Share	83.4%	16.6%	68.2%	31.8%	79.7%	20.3%
Interest Rate	1.12%	1.07%	1.12%	1.09%	1.14%	1.07%
Interbank Rate	1.61%		1.61%		1.54%	
Market Tightness	21.5%		21.1%		26.5%	

The table compares the effect of a risk of withdrawal across the two sectors on the steady-states, with that of the FDIC tax rate on domestic banks.

stitute loans for reserves while domestic banks will choose a lower reserves ratio (comparing columns one to three). In comparison, a change in the risk of foreign banks implies a more significant increase in reserves, more than doubling from column one to two while, the optimal domestic portfolio does not change much. Moreover, we see in column two the foreign share of interbank borrowing increases when the risk is higher, while in column three, it decreases when only the tax for domestic banks is present. Hence bank liquidity risk plays a prominent role in the optimal choice of precautionary reserves and may contribute to the observed trends. Table 8 repeats the exercise of Table 7, given the inclusion of GSEs into the model, albeit the ON RRP rate is equal to the IORB rate, so there is no price differential across outside lending rates of GSEs and banks. We find consistent results, although the reserve ratio shares are smaller because of the lower market tightness. The next table, Table 9, uses counterfactual rates that allow for an additional rent between overnight borrowing and lending to explore the effect of the rate differential on the current estimations.

Table 8: Calibrating a Different Risk of Withdrawals and an FDIC Rate With GSEs

	$\sigma^d = \sigma^f = 0.05,\ tax = 0$		$\sigma^d = 0.05, \sigma^f = 0.113,\ \ tax = 0$		$\sigma^d = \sigma^f = 0.05,\ tax = 0.15\%$	
	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign
Reserve to Assets	10.1%	1.1%	10.2%	3.1%	9.4%	1.1%
Reserve Ratio	11.1%	1.2%	11.2%	3.4%	10.4%	1.2%
Interbank Borrow Share	83.6%	16.4%	70.8%	29.2%	86.5%	13.5%
Total Lending Share	8.3%	1.6%	8.3%	3.8%	7.0%	1.7%
Interest rate	1.12%	1.06%	1.12%	1.07%	1.13%	1.06%
Interbank Rate	1.57%		1.58%		1.57%	
Market Tightness	7.4%		8.4%		9.8%	

The table compares the effect of a risk of withdrawal across the two sectors on the steady-states, with that of the FDIC tax rate on domestic banks.

5.2.4 Long-run effects of a change in each policy tool:

Presence of interbank loans from GSEs- Another important reason for the observed higher reserve ratios after introducing interest on reserve balances is the presence of lending by GSEs. Before September 2013, when the ON RRP rate was unavailable, GSEs alternative overnight-lending rate was zero. The four columns in Table 9 correspond to the Extended model with GSEs facing a second-best rate (called the ON RRP rate) of either zero in the first and second column, or equal to that of the IORB policy rate in the third and fourth column.[29] The rest of the parameters are the same across the simulations: the discount window rate remains at 4%, interest on reserves equals 0.5% on the right part and 0.65% on the left part, and the balance sheet cost associated with the FDIC remains at 0.15%. We compare two different policy rates to show how the assumption of differences in volatility can capture the wide increase in reserves by the foreign sector compared to the domestic, and implications of ON RRP policy on the effects of a change in the IORB rate.

We find that changing the IORB rate from 0.5% to 0.65% has an increasingly bigger impact on the reserve ratios held by foreign banks in the event that ON RRP

[29]Historically, the ON RRP offered is around 10 to 25 basis points lower than the IORB, but this would not change the conclusion of the above simulation.

is zero. Reserves to assets change from 10% to 13% for domestic banks, and from 3.9% to 11% for foreign banks. Also, we find that the rate differential across banks and GSEs implies interbank borrowing will increase with higher IORB, albite the corresponding higher reserve ratios. In turn, market tightness is high and increasing with higher IORB, and therefore the interbank rate does not change. On the other hand, when the ON RRP rate is equal to the IORB rate, higher IORB still implies a moderate increase in reserve ratios, but these are coupled with a decline in interbank borrowing and market tightness, and an increase in the interbank rate.

One important note is that the algorithm of this model may be unable to reach

Table 9: Lending from GSEs in the Interbank Market and the ON RRP Rate

	ON RRP rate is 0%				ON RRP=IORB			
	IORB=0.5%		IORB=0.65%		IORB=0.5%		IORB=0.65%	
	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign
Reserve to Assets	10.0%	3.9%	13.0%	11.0%	9.5%	3.1%	10.4%	5.6%
Reserve Ratio	11.0%	4.2%	14.3%	12.1%	10.4%	3.4%	11.4%	6.2%
Interbank Borrow	0.027	0.013	0.038	0.018	0.012	0.004	0.009	0.003
Interbank Lend	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001
Interbank Borrow Share	68.3%	31.7%	67.9%	32.1%	74.9%	25.1%	76.6%	23.4%
Interbank Lending Share	0.0%	0.0%	0.0%	0.0%	6.9%	3.9%	8.6%	4.8%
Interest rate	1.14%	1.08%	1.14%	1.07%	1.13%	1.07%	1.12%	1.08%
Interbank Rate	1.18%		1.18%		1.57%		1.67%	
Market Tightness	22.4%		28.3%		9.8%		7.3%	

The table compares the steady-state results of adding GSEs with a second-best rate of zero and that equal to the second-best rate of banks.

a steady state given that some specific parameters allow the possibility of arbitrage rent. The failure to reach a steady-state arises when the expected interbank rate is lower than the second-best lending rate. In which case, banks may find it optimal to substitute all loans for borrowing overnight funds against their reserves. In turn, the market tightness will increase, and the interbank rate will increase and eliminate the arbitrage. The problem is that a bank's optimal portfolio switches from arbitraging to not arbitraging at each iteration, given the previous interbank activity and corresponding expected interbank rate, so there is no steady-state in this case. However, the optimal portfolio choice of banks remains. For an example, see Table 15 in appendix A, in which the IORB is set to 1%. Foreign and domestic banks hold only reserves; lending or borrowing against the excess reserves depends on the current market tightness. Nevertheless, the main message remains. Banks may arbitrage and hold more reserves with no ON RRP rate, while an ON RRP equal to the IORB eliminates the arbitrage.

Discount window rate- The following tables, Tables 10- 12, test the changes in other parameters to measure the ceteris-paribus long-run effects of a change in some policy tools. All these tables report steady states of the Extended model (with GSEs) having the ON RRP rate and IORB equal.

Table 10 provides the steady-state solution for the three cases with only a change in the discount window rate from 6% in column one to 4% in column two and 3% in column three. In all cases, the IORB is zero with no FDIC tax, calibrating the rest of the parameters to those specified in Table 3. Results affirm that as the outside cost of borrowing declines, the reserve to assets and the reserve ratios decline. Reserve to asset ratios decline from 9.6% to 4.0% for domestic banks and from 1.6% to 0% for foreign banks. Note that the decline in reserve ratios of domestic banks is coupled with an increase in the total interbank borrowing while that of foreign banks is not. A discount rate of 6% is estimated to be the stigma premium associated with borrowing in the federal funds market. Although admitting that there may be other reasons why banks are observed to hold large excess reserves in an environment of low discount window rates, one implication of this experiment is that lowering the stigma may increase the domestic sector's reliance on the federal funds market to clear their required reserve ratios.

Calibrating different FDIC rates- Table 11 compares how the FDIC tax rate influences the banks' optimal choice using the Extended model. The discount window rate remains at 4%, IORB at 0.5%, and the FDIC tax changes from 0% to 0.15%, and then 0.25% in the three columns, respectively. We find that an independent increase in the FDIC tax is associated with little to no decrease in the

Table 10: Calibrating Different Discount Window Rates

	$r^{dw} = 6\%$		$r^{dw} = 4\%$		$r^{dw} = 3\%$	
	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign
Reserve to Assets	9.6%	1.6%	7.1%	0.1%	4.0%	0.0%
Reserve Ratio	10.5%	1.8%	7.8%	0.1%	4.4%	0.0%
Interbank Borrow Share	70.1%	29.9%	79.6%	20.4%	87.8%	12.2%
Interbank Lending Share	7.2%	3.4%	3.4%	3.0%	1.0%	3.0%
Interest Rate	1.17%	1.08%	1.14%	1.06%	1.10%	1.06%
Interbank Rate	1.80%		1.21%		0.94%	
Market Tightness	10.0%		17.2%		26.2%	

The table compares the steady-state solutions of a lower discount window rate

Table 11: The Effect of the FDIC Tax

	$tax = 0$		$tax = 0.15\%$		$tax = 0.25\%$	
	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign
Reserves to Assets	10.2%	3.2%	9.5%	3.1%	9.1%	3.0%
Reserve Ratio	11.2%	3.5%	10.4%	3.4%	10.0%	3.3%
Interbank Borrow Share	70.6%	29.4%	74.9%	25.1%	76.9%	2213%
Interbank Lending Share	8.3%	3.9%	6.9%	3.9%	6.3%	5.9%
Interest rate	1.17%	1.07%	1.13%	1.07%	1.14%	1.07%
Interbank Rate	1.58%		1.57%		1.57%	
Market Tightness	8.6%		9.8%		10.6%	

The table compares the steady-state solutions of a adding the FDIC tax rate for domestic banks.

reserve ratios of both banks, albeit an even smaller one for foreign banks. The ratios change from 10.2% to 9.1% for domestic banks, and from 3.2% to 3.0%, between the three scenarios for a foreign bank. In addition, policy implies little to no change to the other outcomes in the table. This example exhibits the negligible impact that the FDIC may have on its own, meaning that the observed excess funds held by both sectors relate to other changes such as the large-scale asset purchase that occurred in proximity to the new policy.

Large outflows of funds from the banking system- Another cited claim explaining the large excess reserves held by banks following 2008 is the need for addi-

Table 12: The Effect of a the Withdrawal Distribution

	$\mu = 0$		$\mu = 0.03$		$\mu = 0.05$	
	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign
Reserves to Assets	9.5%	3.1%	11.7%	5.5%	13.2%	6.9%
Reserve Ratio	10.4%	3.4%	12.9%	6.1%	14.5%	7.6%
Interbank Borrow Share	74.9%	25.1%	75.4%	24.6%	75.0%	25.0%
Interbank Lending Share	6.9%	3.9%	6.5%	3.7%	6.2%	3.6%
Interest rate	1.13%	1.07%	1.15%	1.09%	1.16%	1.10%
Interbank Rate	1.57%		1.57%		1.57%	
Market Tightness	9.8%		10.1%		10.2%	

The table compares the steady-state solutions of increasing the probability of an outflow of fund from banks.

tional liquidity. For example, Drechsler et al. (2017) show evidence that depositors pull funds away from banking institutions and into higher-yielding investments following a reduction in the interest rates offered on deposits. Fillat et al. (2018) further document the large outflows of funds from some branches and agencies of U.S. banks with parents in E.U. countries impacted by the sovereign debt crisis. One way to illustrate this is to enforce a negative mean of the withdrawal distribution, increasing the liquidity cost and optimal reserves. Table 12 provides the implications of decreasing the parameter for the mean of the withdrawal distribution μ, with the discount window rate remaining at 4%, the ON RRP rate equal to the IORB rate at 0.5%, and the FDIC set to 0.15%. The result is that the greater the probability of outflow of funds, the higher reserve ratios banks choose. Market tightness does not change, as expected, because outflow is offset with more liquid reserves.

In summary, the central calibration of the model to three different periods does a fair job of matching the interbank share of lending and borrowing data. However, this is only true if a higher discount window rate is calibrated associated with the documented stigma. We find that the foreign banking sector will choose higher reserver ratios mainly because of the higher risk of holding wholesale deposits. However, given some restrictive assumptions on the possible arbitrage from the interbank market, foreign banks and domestic banks may choose to substitute illiquid loans for reserves. In such instances, increasing interest on reserve balances or lowering the average federal funds rate with open market operations would increase this substitution and have contractionary effects. Moreover, the model predicts that the FDIC balance-sheet cost has a relatively small influence on interbanking activity, while the probability distribution and cost of an outflow in withdrawals are influential.

6. Conclusions

Understanding the interbank market is prominent for monetary policy since various monetary policy tools are dependent on how they affect the interbank market environment. Following the financial crisis, this environment has changed, which resulted in large excess reserve accounts, primarily held by foreign banks. Banks that participate in the federal funds market are more responsive to a change in policy (Kashyap and Stein, 2000). Hence, a shift of excess reserves from domestic banks to foreign banks emphasizes the importance of understanding their portfolio choices because those will be the banks that are responding to the policy. Moreover, Cetorelli and Goldberg (2008) show that balance sheets have expanded to include new funding sources given increased international activity. Banks operating abroad can reallocate funds in a liquidity shock occurring either at home or abroad, which means the implication of policy in shaping the environment in which federal funds are exchanged is also crucial because of the role of foreign banks in the global policy transmission.

Following an existing model of Bianchi and Bigio (2014), this paper proposes a general equilibrium model with two banking sectors facing a liquidity mismatch with reallocation of funds possible given an over-the-counter federal funds market for overnight funds while assuming differences in valuations of funds. The market participants include GSEs, U.S. branches of foreign banks, and domestic banks. The heterogeneity between banks stems from the type of deposits of each bank and the FDIC assessment base affecting one type of bank differently than another.

The difference between GSEs and banks is in the policy rates offered by the Fed, which are the interest on reserve balances offered to only banks and the overnight repurchasing rate to GSEs.

We have shown that the inclusion of heterogeneous agents in a model of the interbank market has policy implications. For example, an increase in reserves of domestic banks can be attributed to the expected moments of deposit withdrawals and the stigma associated with clearing transactions in the discount window facility. Also, an increase in the IORB rate has a more considerable impact on the excess reserves held by foreign banks than domestic banks. This result holds when there are no differences in the balance-sheet costs of the two sectors but more so when accounting for these differences.

An important conclusion of the model is that the high risk of withdrawals of foreign banks is sufficient to motive significant increases in reserves, and thus could be attributed to the trends we observe because U.S. branches of foreign banks may only hold wholesale deposits. The implication is that open market operation to lower the federal funds rate may be dampened by the foreign sector increasing reserves at times when the risk of withdrawals is high.

The current model is solved for the steady states. One limitation in the scope of these results is that they lack the impact of large-scale assets purchases widely used by monetary policy. Such policy significantly impacts the interbank market tightness as it becomes satiated in the availability of overnight funds and corresponds to low interbank rates. The implication of this on the current results could be significant. For example, when the market is satiated and the interbank bargaining power of lenders is small, the differential across agents outside rates could result in significant arbitrage gains from borrowing low in the federal funds market and lending for the IORB rate. We find that the model fails to find a steady-state in this case quite precisely because of the arbitrage borrowing- as banks optimize by holding extensive overnight borrowing, the market becomes tight once again. However, the policy implications remain. Reducing the differences across agents' valuations of overnight funds is optimal for the conduct of monetary policy.

References

Afonso, Gara and Ricardo Lagos, "Trade Dynamics in the Market for Federal Funds," *Econometrica*, 2015, *83* (1), 263–313.

_ , Roc Armenter, and Benjamin Lester, "A Model of the Federal Funds Market: Yesterday, Today, and Tomorrow," *Review of Economic Dynamics*, 2019, *33*, 177–204.

Armantier, Olivier, Eric Ghysels, Asani Sarkar, and Jeffrey Shrader, "Discount Window Stigma During the 2007–2008 Financial Crisis," *Journal of Financial Economics*, 2015, *118* (2), 317–335.

Armenter, Roc and Benjamin R. Lester, "Excess Reserves and Monetary Policy Implementation," *Review of Economic Dynamics*, 2016, *23*, 212–235.

Atkeson, Andrew G, Andrea L Eisfeldt, and Pierre-Olivier Weill, "Entry and Exit in OTC Derivatives Markets," *Econometrica*, 2015, *83* (6), 2231–2292.

Bech, Morten L and Elizabeth Klee, "The Mechanics of a Graceful Exit: Interest on Reserves and Segmentation in the Federal Funds Market," *Journal of Monetary Economics*, 2011, *58* (5), 415–431.

Berlin, Mitchell et al., "New Rules for Foreign Banks: What's at Stake?," *Business Review Q*, 2015, *1*, 1–10.

Bernanke, Ben, "The Federal Funds Rate and the Channels of Monetary Transmission," Technical Report, National Bureau of Economic Research 1990.

Bernanke, Ben S and Mark Gertler, "Inside the Black Box: the Credit Channel of Monetary Policy Transmission," *Journal of Economic Perspectives*, 1995, *9* (4), 27–48.

Bianchi, Javier and Saki Bigio, "Banks, Liquidity Management and Monetary Policy," Working Paper 20490, National Bureau of Economic Research 2014.

_ and _ , "Settlement Frictions and Portfolio Theory," Technical Report, Mimeo, Federal Reserve Bank of Minneapolis 2017.

Cetorelli, Nicola and Linda S Goldberg, "Banking Globalization, Monetary Transmission and the Lending Channel," Working Paper 14101, National Bureau of Economic Research 2008.

Cochrane, John H, "Monetary Policy with Interest on Reserves," *Journal of Economic Dynamics and Control*, 2014, *49*, 74–108.

Committee, Basel et al., "Basel III: A Global Regulatory Framework for More Resilient Banks and Banking Systems," *Basel Committee on Banking Supervision, Basel*, 2010.

Craig, Ben R and Sara Millington, "The Federal Funds Market Since the Financial Crisis," *Economic Commentary*, 2017, pp. 2017–07.

DiSalvo, James, "How Foreign Banks Changed After Dodd–Frank," *Federal Reserve Bank of Philadelphia Economic Insights (USA)*, 2019, *3*, 1–6.

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl, "The Deposits Channel of Monetary Policy," *The Quarterly Journal of Economics*, 2017, *132* (4), 1819–1876.

Duffie, Darrell, Nicolae Gârleanu, and Lasse Heje Pedersen, "Valuation in Over-the-counter Markets," *The Review of Financial Studies*, 2007, *20* (6), 1865–1900.

Ennis, Huberto M, "A Simple General Equilibrium Model of Large Excess Reserves," *The Federal Reserve Bank of Richmond Working Paper WP*, 2014, pp. 14–14.

_ , Alexander L Wolman et al., "Large Excess Reserves in the United States: A View from the Cross-section of Banks," *International Journal of Central Banking*, 2015, *11* (1), 251–89.

Fillat, Jose, Arthur Smith, Stefania Garetto et al., "To Branch or not to Branch? A Quantitative Evaluation of the Consequences of Global Banks' Organization," in "in" 2018.

Hanson, Samuel G, Andrei Shleifer, Jeremy C Stein, and Robert W Vishny, "Banks as Patient Fixed-income Investors," *Journal of Financial Economics*, 2015, *117* (3), 449–469.

Hein, Scott E, Timothy W Koch, and Chrislain Nounamo, "Moving FDIC Insurance to an Asset-based Assessment System: Evidence from the Special Assessment of 2009," *Journal of Economics and Business*, 2012, *64* (1), 24–36.

Hornstein, Andreas, "Monetary Policy with Interest on Reserves," *FRB Richmond Economic Quarterly*, 2010, *96* (2), 153–177.

Ireland, Peter N, "The Macroeconomic Effects of Interest on Reserves," *Macroeconomic Dynamics*, 2014, *18* (06), 1271–1312.

Kashyap, Anil K and Jeremy C Stein, "The Impact of Monetary Policy on Bank Balance Sheets," in "Carnegie-Rochester Conference Series on Public Policy," Vol. 42 Elsevier 1995, pp. 151–195.

_ and _ , "What Do a Million Observations on Banks Say About the Transmission of Monetary Policy?," *American Economic Review*, 2000, pp. 407–428.

_ and _ , "The Optimal Conduct of Monetary Policy with Interest on Reserves," *American Economic Journal: Macroeconomics*, 2012, *4* (1), 266–282.

Kotok, David, "Scylla and Charybdis, The FDIC and the Federal Reserve,," Technical Report 2011.

Kreicher, Lawrence, R McCauley, and Patrick McGuire, "The 2011 FDIC Assessment on Banks Managed Liabilities: Interest Rate and Balance-sheet Responses," *Taxation and Regulation of the Financial Sector*, 2014, p. 333.

Lenel, Moritz, Monika Piazzesi, and Martin Schneider, "The Short Rate Disconnect in a Monetary Economy," *Journal of Monetary Economics*, 2019, *106*, 59–77.

Lester, Benjamin et al., "Implementing Monetary Policy in a Changing Federal Funds Market," *Economic Insights*, 2019, *4* (2), 15–20.

Poole, William, "Commercial Bank Reserve Management in a Stochastic Model: Implications for Monetary Policy," *The Journal of Finance*, 1968, *23* (5), 769–791.

Whalen, Richard Christopher, "What is a Core Deposit and Why Does it Matter? Legislative and Regulatory Actions Regarding FDIC-insured Bank Deposits Pursuant to the Dodd-Frank Act," *Networks Financial Institute Working Paper*, 2011, (2011-WP), 14.

A. Additional Steady-State Tables

The following Tables report the steady-state solution for the central result in which no stigma premium is attributed to borrowing from the discount window facility. In Table 13, the change in reserves ratio between the three periods is much lower than the conclusion with the discount window rate with the stigma as in the main text. Other than that, we see the federal funds rate is closer than that observed during these periods, although not as low. Also, note that even though the discount window rate increase from 0.5% to 0.75%, reserve ratios reduce to zero, moving from the second to the third column once the FDIC policy is introduced. This result counters the hypothesis that an FDIC assessment associated with a higher balance-sheet cost for domestic banks will increase the opportunity cost of reserves. The conclusion holds for Table 14 that reports an example with a low discount window rate with the extended model.

Table 13: Alternative Steady-state Solution Given a Lower Discount Window Rate as Reported by the Federal Reserves

	2008,Q3		2008,Q4		2011,Q2		2011,Q3	
	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign
Reserve to Assets	8.6002	0.5312	9.7128	2.2982	9.5405	1.8984	9.0373	3.5532
Reserve Ratio	9.4602	0.5843	10.6841	2.5281	10.4948	2.0883	9.9410	3.9087
Interbank Borrow	0.0132	0.0048	0.0109	0.0044	0.0116	0.0047	0.0136	0.0037
Interbank Lend	0.0115	0.0064	0.0105	0.0051	0.0110	0.0053	0.0104	0.0070
Discount Window	0.0046	0.0017	0.0015	0.0006	0.0016	0.0007	0.0019	0.0005
Interbank Borrow Share	73.3614	26.6386	71.1244	28.8756	71.1516	28.8484	78.6471	21.3529
Interbank Lending Share	64.2450	35.7550	67.3727	32.6273	67.2312	32.7688	59.8446	40.1554
DW/Reserves	0.5674	18.9083	0.1682	1.6354	0.1813	2.0939	0.2249	0.8802
Interest rate	2.6465	2.4320	1.6479	1.5842	1.1389	1.0701	1.1499	1.0824
Market Tightness	54.2%		39.6%		41.8%		44.1%	
Federal funds rate	2.48		1.78		1.23		1.22	

The last table shows two iterations of one of the simulations in which there is no steady-state. We see that increasing the IORB slightly above 1% will result in

Table 14: Alternative Steady-state Solution Given a Lower Discount Window Rate as Reported by the Federal Reserves, Including GSEs

	2008,Q3		2008,Q4		2010,Q4		2011,Q1	
	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign	Domestic	Foreign
Reserve to Assets	0.1937	0.1002	0.3124	0.1189	4.6777	0.0009	4.1563	0.4430
Reserve Ratio	0.2131	0.1103	0.3436	0.1308	5.1457	0.0010	4.5719	0.4874
Interbank Borrow	0.0733	0.0060	0.0723	0.0059	0.0388	0.0060	0.0425	0.0057
Interbank Lend	0.0001	0.0024	0.0001	0.0024	0.0006	0.0014	0.0005	0.0016
Discount Window	0.0102	0.0008	0.0101	0.0008	0.0054	0.0008	0.0059	0.0008
Interbank Borrow Share	92.4656	7.5344	92.4004	7.5996	86.5429	13.4571	88.1746	11.8254
Interbank Lending Share	0.1277	3.0659	0.1356	3.0677	1.2913	2.9929	1.0211	3.1507
DW/Reserves	56.5900	50.4634	34.5572	42.2915	1.2387	5665.2607	1.5278	10.9082
Interest rate	2.4376	2.4254	1.5743	1.5691	1.0975	1.0564	1.1007	1.0610
Market Tightness	35.8%		35.9%		24.3%		25.7%	
Federal funds rate	1.52		1.22		0.94		0.95	

this extensive arbitrage by both foreign and domestic banks who optimally choose to issue only interbank loans, so the reserve to deposit ratio is 100%. A problem with the model given such specification arises because it fails to find a steady-state in the interbank market. It happens because when optimal to borrow against all reserves, the market becomes tight as borrowing orders exceed lending orders. Tightness implies a higher expected federal funds rate that will result in interbank lending instead. If banks find it optimal to lend all excess reserves, the market becomes satiated again, and rates decline. The limitation of the model is that the algorithm jumps from a tight to a satiated market and never reaches a steady state.

Table 15: Presence of Arbitrage with No Steady-state Solution (Two Consecutive Iterations

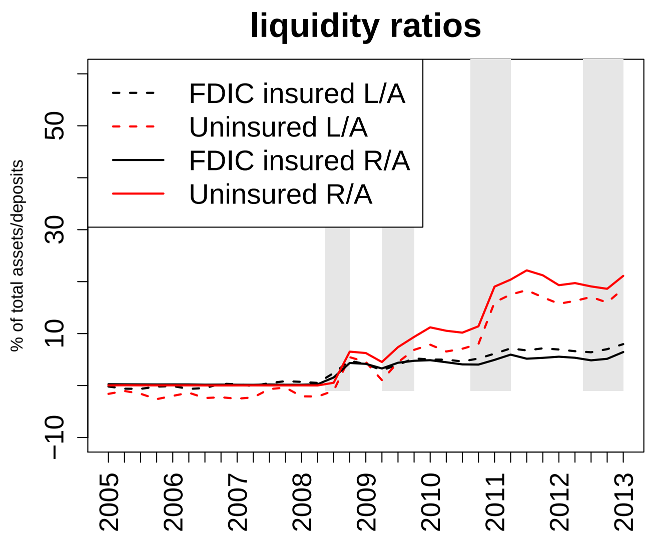
	Domestic	Foreign	Domestic	Foreign
Reserves to Assets	90.86%	90.83%	90.86%	90.83%
Reserve Ratio	99.95%	99.93%	99.95%	99.93%
Interbank Borrowing Share	NaN	NaN	100%	0
Interbank Lending Share	67.3%	13.21%	0%	40.52%
Interbank Borrow	0	0	0.671	0
Interbank Lend	0.141	0.029	0	0.028
Discount Window	0	0	0.094	0
Interest Rate	1.12%	1.19%	1.11%	1.17%
Interbank Rate	1.57%		2.14%	
Market Tightness	23.3%		24.6%	

B. Data

Figure 6, plots the liquidity to assets of each sector (the dotted lines) and reserve to assets for each sector (the solid lines). The red line corresponds to Uninsured U.S. branches of foreign banks, and the black line corresponds to the domestically chartered insured banks. From the comparison we see a slight substitution of reserves for foreign banks following the introduction of interest on excess reserves, but not for domestic banks.

Some of the macroeconomic variables used in this paper have complied from the Federal Financial Institutions Examination Council (FFIEC) quarterly filings. FFIEC 031, Reports of Condition and Income (also known as the Call Reports) for domestically chartered banks, and FFIEC 002, the Report of Assets and Liabilities for branches and agencies of foreign banking organizations. Other macroeconomic variables listed below are obtained from the Federal Reserve Bank of St. Louis Economic Research Database (FRED) or the Federal Reserve Bank of New York. The original data sources for each series are collected by the Board of Governors of the Federal Reserve System (US).

- the share of interbank lending:



  - Board of Governors of the Federal Reserve System (US), Fed Funds Lending (2006-2012), retrieved from Federal Reserve Bank of New York;

    https://www.newyorkfed.org/fed-funds-lending

- The share of GSEs overnight loan orders to total assets:
  - Board of Governors of the Federal Reserve System (US), Government-Sponsored Enterprises; Total Assets Held by FHLB (Balance Sheet), Level [BOGZ1FL404090430Q], retrieved from FRED, Federal Reserve Bank of St. Louis;

    https://fred.stlouisfed.org/series/BOGZ1FL404090430Q

    Board of Governors of the Federal Reserve System (US), Government-Sponsored Enterprises; Total Assets (Balance Sheet), Level [BOGZ1FL404090405Q], retrieved from FRED, Federal Reserve Bank of St. Louis;

    https://fred.stlouisfed.org/series/BOGZ1FL404090405Q

- the share of interbank borrowing:

  - Board of Governors of the Federal Reserve System (US), Fed Funds Borrowing (2006-2012), retrieved from Federal Reserve Bank of New York;

    https://www.newyorkfed.org/fed-funds-borrowing

- the share of discount window loan to reserves
  - Board of Governors of the Federal Reserve System (US), Total Borrowings of Depository Institutions from the Federal Reserve excluding Term Auction Credit (DISCONTINUED) [DISCBORR], retrieved from FRED, Federal Reserve Bank of St. Louis;

    https://fred.stlouisfed.org/series/DISCBORR

  - Board of Governors of the Federal Reserve System (US), Reserves of Depository Institutions: Total [TOTRESNS], retrieved from FRED, Federal Reserve Bank of St. Louis;

    https://fred.stlouisfed.org/series/TOTRESNS

- the interest on discount window loans:
  - Board of Governors of the Federal Reserve System (US), Discount Window Primary Credit Rate [DPCREDIT], retrieved from FRED, Federal Reserve Bank of St. Louis;

    https://fred.stlouisfed.org/series/DPCREDIT

- the interest on reserves:
  - Board of Governors of the Federal Reserve System (US), Interest Rate on Excess Reserves [IOER], retrieved from FRED, Federal Reserve Bank of St. Louis;

https://fred.stlouisfed.org/series/IOER

- the effective federal funds rate:
  - Board of Governors of the Federal Reserve System (US), Federal Funds Effective Rate [FEDFUNDS], retrieved from FRED, Federal Reserve Bank of St. Louis;

https://fred.stlouisfed.org/series/FEDFUNDS

C. Four Cases

Bargaining problem with two banks: a foreign bank and a domestic bank. At the start of each balancing stage, each type of bank decides to borrow or lend overnight funds in the interbank market. Each having a different outside option available to it when bargaining for r_t^{ff}. There is a tax τ on reserves held by domestic banks. So while the outside option of overnight lending for the foreign bank faces is r_t^{ior}, the outside option facing the domestic bank equals $r_t^{ior} - \tau$. Because of these asymmetries, the Nash Bargaining problem takes different forms depending on which type of bank is lending and borrowing. Where it is without saying that, in the case of only two banks, we get that in periods that both choose to borrower overnight funds or both choose to lend overnight funds, there is no trade between them in that period, and each resolves to the outside option available to it. As we will see, the choice of being a lender or a borrower could be independent of the actual amount of reserves one bank has and will lead to different bargaining solutions. As in Bianchi and Bigio (2014), I assume that banks will always prefer to settle reserves in the interbank market first. The Nash Bargaining problem for lenders and borrowers has a bargaining rate $\varepsilon \in [0,1]$ for each party's bargaining power. With no loss in generality, equal bargaining power rates are assumed for simplicity[30].

The bargaining problem of a unit of reserves has a first-order condition that provides an implicit solution for the federal funds rate in each of the four different cases, as shown below.

Case 1: The foreign bank has excess reserves while the domestic bank has a reserves deficit

$$\max_{r_t^{ff}} \left(m_b r_t^{dw} - m_b r_t^{ff}\right)^{\varepsilon} \left(m_l r_t^{ff} - m_l r_t^{ior}\right)^{1-\varepsilon}$$

[30] In the model, it is a function of the probability of matching in the market. So that if the probability of matching a borrowing order is low while matching a lending order is high, the bargaining power is skewed towards lenders in determining the federal funds rate.

Solving for the first order conditions we get that

$$r_t^{ff} = (1-\varepsilon)r_t^{dw} + \varepsilon(r_t^{ior}) \tag{44}$$

Case 2: The domestic Bank has excess reserves while the foreign Bank has a reserves deficit

$$\max_{r_t^{ff}} \left(m_b r_t^{dw} - m_b r_t^{ff}\right)^{\varepsilon} \left(m_l r_t^{ff} - m_l(r_t^{ior} - \tau)\right)^{1-\varepsilon}$$

Solving for the first order conditions we get that

$$r_t^{ff} = (1-\varepsilon)r_t^{dw} + \varepsilon(r_t^{ior} - \tau) \tag{45}$$

The asymmetry in the marginal benefit of holding excess reserves between the two banks assures a violation of the non-arbitrage condition when the federal funds rate falls between $r_t^{ior} - \tau$, and r_t^{ior}. A foreign bank can borrow additional overnight funds at $r_t^{ff} \leq r_t^{ior}$ to lend to the Fed at r_t^{ior} for a profit, suggesting that a foreign bank is always a borrower if it can bargain the federal funds rate such that $r^{ior} - \tau \leq r^{ff} \leq r^{ior}$, even if it holds excess reserves.

Case 3: Both banks have excess reserves (the domestic bank is a lender and the foreign bank a borrower)

A foreign bank will have the outside option r_t^{ior}, which is the opportunity cost of not borrowing additional funds. While the domestic bank lends with a lower outside option of $r_t^{ior} - \tau$

$$\max_{r_t^{ff}} \left(m_b r_t^{ior} - m_b r_t^{ff}\right)^{\varepsilon} \left(m_l r_t^{ff} - m_l(r_t^{ior} - \tau)\right)^{1-\varepsilon}$$

Solving for the first order conditions we get that

$$r_t^{ff} = (1-\varepsilon)r_t^{ior} + \varepsilon(r_t^{ior} - \tau),$$

or

$$r_t^{ff} = r_t^{ior} - \varepsilon\tau. \tag{46}$$

Case 4: Both banks have a deficit (domestic bank is a lender and foreign bank a borrower)

$$\max_{r_t^{ff}} \left(m_b r_t^{dw} - m_b r_t^{ff} \right)^{\varepsilon} \left(m_l r_t^{ff} - m_l r_t^{dw} \right)^{1-\varepsilon}$$

Solving for the first order conditions we get that

$$r_t^{ff} = r_t^{dw} \tag{47}$$

D. Steady-State with Two Banks

- step 1:

  1. Guess the initial (perfectly competitive profit-maximizing) stead-state real returns on illiquid loans r^D_{ss}, and r^F_{ss}, for the domestic sector and the foreign sector.
  2. Guess the initial steady-state market tightness for overnight funds, θ_{ss}.
  3. Guess the initial stead-state probabilities that a borrower of overnight funds is borrowing from a domestic bank, γ^-_D, and that a lender of overnight funds is lending to a domestic bank, γ^+_D (where $\gamma^-_F = (1-\gamma^+_D)$, and $\gamma^+_F = (1-\gamma^-_D)$, are the analog probabilities that an order is met with a foreign counterpart).

- step 2:

  1. Based on the market tightness and monetary policy, the trading probabilities γ^+_{ss}, and γ^-_{ss} and the bargaining power ϕ_{ss} are determined. Based on the initial guess for the probability of facing a domestic or a foreign bank in the market, and γ^+_{ss}, and γ^-_{ss} the corresponding expected Nash-bargaining, federal funds rate is computed and with it the corresponding liquidity cost/benefit of having reserves deficit/excess.

$$\gamma^+_{ss} = \left\{\begin{matrix} 1-e^{-\lambda} & \text{if } \theta_{ss} \geq 1 \\ \theta_{ss}(1-e^{-\lambda}) & \text{otherwise} \end{matrix}\right\}$$

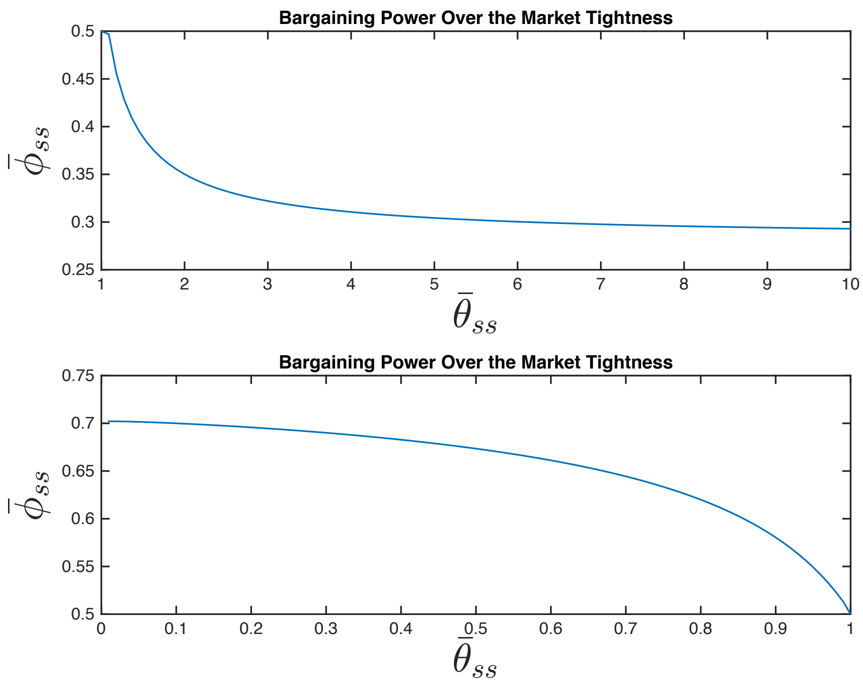
$$\gamma^-_{ss} = \left\{\begin{matrix} 1-e^{-\lambda} & \text{if } \theta_{ss} \leq 1 \\ \theta^{-1}_{ss}(1-e^{-\lambda}) & \text{otherwise} \end{matrix}\right\}$$

$$\bar{\theta}_{ss} = \left\{\begin{matrix} 1+(1+e^{\lambda})(\theta_{ss}-1) & \text{if } \theta_{ss} > 1 \\ (1+(\theta^{-1}_{ss}-1)e^{\lambda})^{-(e^{-\lambda}+\bar{\phi})} & \text{otherwise,} \end{matrix}\right\}$$

given λ the market friction parameter and

$$\phi_{ss} = \begin{cases} \left(\left(\dfrac{\bar{\theta_{ss}}}{\theta_{ss}}\right)^{\bar{\phi}} - 1\right)\dfrac{\theta_{ss}}{\theta_{ss}-1}(e^{\lambda}-1)^{-1} & \text{if } \theta_{ss} > 1 \\ \bar{\phi} & \text{if } \theta_{ss} = 1 \\ \left(\dfrac{1}{1-\theta_{ss}}\right)\left(\left(\dfrac{\bar{\theta_{ss}}}{\theta_{ss}}\right)^{\bar{\phi}} - 1\right)(e^{\lambda}-1)^{-1}(\bar{\phi} + e^{\bar{\phi}-\lambda}) & \text{otherwise.} \end{cases}$$

A parameter of $\bar{\phi} = 0.5$ implies an equal bargaining parameter when the



lending orders and borrowing orders are equal. With the current specification[31], the bargaining power function is plotted in the figure above. We see that as the market is tighter bargaining parameter converters to 0.3, and as the market get satiated with more funds and θ_{ss} is closer to zero, ϕ_{ss} gets closer to 0.7.

The associated liquidity cost for a domestic bank that needs to borrow

[31] along with eta could be scale differently to change the boundaries of the equation to be closer to 0 and 1- but this results in algorithm in the single representative bank model to fail and converge to steady-state for some certain conditions.

reserves equals:

$$\chi_D^- = \begin{cases} r^{dw} - r^{ior} + tax & \text{if } r^{dw} \leq r^{ff} \\ \gamma_{ss}^-(r^{ff} - (r^{ior} - tax)) + (1 - \gamma_{ss}^-)(r^{dw} - (r^{ior} - tax)) & \text{otherwise,} \end{cases}$$

with the expected federal funds rate for the domestic borrower is the solution to the Nash bargaining problem such that

$$r_1^{ff} = (1 - \phi_{ss})r^{dw} + \phi_{ss}(\gamma_{D}^-{}_{ss}(r^{ior} - tax) + \gamma_{F}^-{}_{ss}r^{ior}),$$

The liquidity benefit of having excess reserves is

$$\chi_D^+ = \gamma_{ss}^+(r^{ff} - (r^{ior} - tax)),$$

with the expected federal funds rate for the domestic lender equal to

$$r_2^{ff} = (1 - \phi_{ss})(\gamma_{ssD}^+ r^{dw} + \gamma_{F}^+{}_{ss}r^{dw}) + \phi_{ss}(r^{ior} - tax).$$

The liquidity cost for a foreign bank to borrower reserves equals:

$$\chi_F^- = \gamma_{ss}^-(r^{ff} - (r^{ior} - tax)) + (1 - \gamma_{ss}^-)(r^{dw} - r^{ior})$$

with the expected federal funds rate for a foreign borrower

$$r_3^{ff} = (1 - \phi_{ss})r^{dw} + \phi_{ss}(\gamma_{D}^-{}_{ss}(r^{ior} - tax) + \gamma_{F}^-{}_{ss}r^{ior}).$$

The liquidity benefit of having excess reserves depends on if the foreign bank is a lender or an arbitrageur. The arbitrage of borrowing from the interbank market and lending to the fed is equal to

$$arbitrage = \gamma_{ss}^-(r^{ior} - r_{ab}^{ff})$$

with the arbitrageur's expected federal funds rate,

$$r_{ab}^{ff} = (1-\phi_{ss})r^{ior} + \phi_{ss}(\gamma_{D\,ss}^{-}(r^{ior} - tax) + \gamma_{F\,ss}^{-}r^{ior})$$

A foreign bank will choose to arbitrage if $lend < arbitrage$, otherwise it will lend its excess reserves. In which case, the expected lending rate is

$$r_4^{ff} = (1-\phi_{ss})(\gamma_{D\,ss}^{+}r^{dw} + \gamma_{F\,ss}^{+}r^{dw}) + \phi_{ss}r^{ior}$$

A lenders expected interest rate and return on loan is:

$$lend = \gamma_{ss}^{+}(r^{ff} - r^{ior})$$

The resulting liquidity benefit for a foreign bank is

$$\chi_F^{+} = \left\{ \begin{matrix} \gamma_{ss}^{+}(r^{ff} - r^{ior}) & \text{if } lend \geq arbitrage \\ \gamma_{ss}^{-}(r^{ior} - r_{ab}^{ff}) & \text{otherwise,} \end{matrix} \right\}$$

2. Solve foreign and domestic banks' optimization problem with initial guess for reserves $\bar{m}_{ss}^{D}$, and $\bar{m}_{ss}^{F}$ and for deposits, $\bar{d}_{ss}^{D}$, and $\bar{d}_{ss}^{F}$.
Domestic surplus/ deficit is defined as:

$$s^D(\omega) \equiv \bar{m}^D + r^d/r^{ior}\omega\bar{d}^D - \rho\bar{d}^D(1+\omega).$$

$$\omega_D^{*} = (\rho - (\bar{m}_{ss}^{D}/\bar{d}_{ss}^{D})/(\frac{r^d}{r^{ior}} - \rho)$$

is the ω at which a bank has no surplus and no deficit. Then the bank will choose $\bar{m}^D_{ss}$, and $\bar{d}^D_{ss}$ to maximize

$$\Omega^D_{ss} = \max_{\bar{d}^D \in [0, \frac{\kappa}{1-\kappa}], \bar{m}^D \in [0, 1+\bar{d}^D]} \int_{\omega^*}^{\bar{\omega}_D} ln\Big[r^D_{ss} + (r^{ior} - tax - r^D_{ss})\bar{m}^D + (r^D_{ss} - r^d)\bar{d}^D + \chi^+_D max\big(s(\omega), 0\big)\Big]\Phi(\omega)d\omega + \int_{\underline{\omega}}^{\omega^*} \Big[ln\Big\{r^D_{ss} + (r^{ior} - tax - r^D_{ss})\bar{m}^D + (r^D_{ss} - r^d)\bar{d}^D - \chi^-_D max\big(-s(\omega), 0\big)\Big]\Phi(\omega)d\omega$$

where

$$\Phi(\cdot) = \frac{e^{-\frac{(\omega-\mu)}{sd^D}}}{sd^D\left(1 + e^{-\frac{(\omega-\mu)}{sd^D}}\right)^2}$$

is the pdf for the logistic distribution, and

$$\bar{b}^D_{ss} = 1 + \bar{d}^D_{ss} - \bar{m}^D_{ss}$$

is the choice of domestic loans given the bank's balance sheet constraint. Foreign surplus/ deficit is defined as:

$$s(\omega)^F \equiv \bar{m}^F + r^d/r^{ior}\omega\bar{d}^F,$$

and the threshold

$$\omega^*_F = \frac{-\bar{m}^D_{ss}}{\bar{d}^D_{ss}} / \frac{r^d}{r^{ior}}$$

the bank will choose $\bar{m}^F_{ss}$, and $\bar{d}^F_{ss}$ to maximize

$$\Omega^F_{ss} = \max_{\bar{d}^F \in [0,1], \bar{m}^F \in [0, 1+\bar{d}^F]} \int_{\omega^*}^{\bar{\omega}} ln\Big[r^F_{ss} + (r^{ior} - r^F_{ss})\bar{m}^F + (r^F_{ss} - r^d)\bar{d}^F + \chi^+_F max\big(s(\omega), 0\big)\Big]\Phi(\omega)d\omega + \int_{\underline{\omega}}^{\omega^*} \Big[ln\Big\{r^F_{ss} + (r^{ior} - r^F_{ss})\bar{m}^F + (r^F_{ss} - r^d)\bar{d}^F - \chi^-_F max\big(-s(\omega), 0\big)\Big]\Phi(\omega)d\omega$$

with a distribution for ω common to all foreign bank

$$\Phi(\cdot) = \frac{e^{-\frac{(\omega-\mu)}{sd^F}}}{sd^F\left(1+e^{-\frac{(\omega-\mu)}{sd^F}}\right)^2},$$

In which the two sectors have a different standard deviation for withdrawals (foreign bank has a higher risk).

$$\bar{b}^F_{ss} = 1 + \bar{d}^F_{ss} - \bar{m}^F_{ss}$$

is the steady-state choice of private sector loans issued by the foreign bank.
Given log utility we get that steady-state dividends follows

$$c_{ss} = 1 - \beta.$$

This is the same for both domestic and foreign banks and independent from the optimization problem

- step 3: Check whether banks' policies are consistent with steady state, if not adjust guess for i^D_{ss}, i^F_{ss}, and θ_{ss}

  1. Measure the surplus and deficit of each sector:

$$D^+ = \int_{\omega^*_D}^{\bar{\omega_D}} s^D(\omega)\Phi(\omega,\sigma^D)d\omega,$$

$$D^- = \int_{\underline{\omega}_D}^{\omega^*_D} -s^D(\omega)\Phi(\omega,\sigma^D),$$

With foreign banks arbitraging we have that when $i^{ior} > i^{ff}_{ab}$ the measure of surplus for this sector is added to the amount of borrowing orders.

$$F^{+} = \begin{Bmatrix} \int_{\omega_F^*}^{\bar{\omega_F}} s^F(\omega)\Phi(\omega,\sigma^F)d\omega, & \text{if } i^{ior} \leq i^{ff}_{ab} \\ 0 & \text{otherwise} \end{Bmatrix}$$

and

$$F_1^{-} = \begin{Bmatrix} \int_{\omega_F^*}^{\bar{\omega_F}} s^F(\omega)\Phi(\omega,\sigma^F)d\omega & \text{if } i^{ior} \leq i^{ff}_{ab} & \\ & 0 & \text{otherwise} \end{Bmatrix}$$

$$F_2^{-} = \int_{\bar{\omega}_F}^{\omega_F^*} -s^F(\omega)\Phi(\omega,\sigma^F)d\omega,$$

2. Compute the aggregate lending orders

$$M^{+} = shareD^{+} + (1-share)F^{+},$$

and the aggregate borrowing orders

$$M^{-} = shareD^{-} + (1-share)(F_1^{-} + F_2^{-})$$

3. Measure the average federal funds rate

$$\bar{r}^{ff} = \phi_{ss}(w^2 r_2^{ff} + w^4 r_4^{ff}) + (1-\phi_{ss})(w^1 r_1^{ff} + w^3 r_3^{ff} + w^5 r_{ab}^{ff})$$

with the weights of the different federal funds rate depending on the masses of each type of borrowers and masses of each type of lenders such that

$$w^2 = share\left(\frac{\gamma^{+}D^{+}}{M^{+}}\right)$$

$$w^4 = (1-share)\left(\frac{\gamma^{+}F^{+}}{M^{+}}\right)$$

$$w^1 = share\left(\frac{\gamma^- D^-}{M^-}\right)$$

$$w^3 = (1 - share)\left(\frac{\gamma^- F_2^-}{M^-}\right)$$

$$w5 = (1 - share)\left(\frac{\gamma^- F_1^-}{M^-}\right)$$

4. Measure the equity growth of each sector

$$\begin{aligned} D_{equity} =& \beta(r^D_{ss}\bar{b}^D_{ss} + (r^{ior} - tax)\bar{m}^D_{ss} - r^d\bar{d}^D_{ss} \\ & - (r^{dw} - r^{ior} + tax)(1 - \gamma^- ss)D^- + (\bar{r}^{ff} - r^{ior} + tax)(\gamma^+_{ss}D^+ - \gamma^-_{ss}D^-)) \end{aligned}$$

$$\begin{aligned} F_{equity} =& \beta(r^F_{ss}\bar{b}^F_{ss} + r^{ior}\bar{m}^F_{ss} - r^d r\bar{d}^F_{ss} \\ & - (r^{dw} - r^{ior})(1 - \gamma^- ss)F^- + (\bar{r}^{ff} - r^{ior})(\gamma^+_{ss}F^+ - \gamma^-_{ss}F^-)) \end{aligned}$$

and the aggregate equity growth of the whole banking based on the initial size of each sector follows

$$E^g_{ss} = shareD_{equity} + (1 - share)F_{equity}$$

The market clearing condition for the interbank market follows

$$\gamma^+_{ss}(shareD^+ + (1 - share)F^+) = \gamma^-_{ss}(shareD^- + (1 - share)F^-)$$

The federal funds market clearing simply states that aggregate lending orders equal the aggregate borrowing orders matched in the market. By substitution we that the aggregate equity growth becomes

$$
\begin{aligned}
E^g_{ss} =& \beta[share(r^D_{ss}\bar{b}^D_{ss} + (r^{ior} - tax)\bar{m}^D_{ss} - r^d\bar{d}^D_{ss} - (r^{dw} - r^{ior} + tax)(1 - \gamma^- ss)D^- \\
& + tax * (\gamma^+_{ss}D^+ - \gamma^-_{ss}D^-)) + (1 - share)(r^F_{ss}\bar{b}^F_{ss} + r^{ior}\bar{m}^F_{ss} - r^d r\bar{d}^F_{ss} \\
& - (r^{dw} - r^{ior})(1 - \gamma^- ss)F^-)]
\end{aligned}
$$

5. The updated probabilities that a lending order is matched is γ^+. The conditional probability that this lending order is matched with a domestic bank given a match has occurred is simply the ratio of domestic banks to foreign banks which are borrowing. So that

$$\gamma^+_D = shareD^-/M^-$$

and

$$\gamma^-_D = shareD^+/M^+.$$

Similarly, the updated probabilities that a borrowing order is matched is γ^- and the conditional probabilities follow

$$\gamma^-_F = (1 - share)(F^-_1 + F^-_2)/M^-$$

$$\gamma^+_F = (1 - share)F^+/M^+.$$

Lastly, the update market tightness equal to

$$\tilde{\theta} = M^-/M^+$$

6. Update rule:

– If $|\tilde{\theta} - \theta_{ss}|$ larger than tolerance adjust guess for θ_{ss} such that

$$\theta_{ss} = .5\theta_{ss} + .5\tilde{\theta}$$

– if $|\tilde{\theta} - \theta|$ is less than tolerance, but $|E_{ss} - 1| > tol$ check if $|D_{equity} - 1| > tol$ or $|F_{equity} - 1| > tol$, and change the guess for the interest rate on loans appropriately by decreasing/increasing the interest rates if equity is greater/lower than one (i.e change i^F_{ss} if foreign equity is different form one, and change i^D_{ss} if domestic equity needs to be adjusted).

Introducing GSEs:

- step 1:

1. Set an exogenous mass of GSEs lending $G^+ \in [0, 11]$, as the unit-equity GSEs' amount of lending orders, and a the size of GSEs relative to banks (for example if GSEs size is twice as large as all banks a=2). The relative mass of foreign banks, domestic banks, and GSEs is redefined as: $\bar{a} = a/(1+a)$, $sh\bar{a}re = share/(1+a)$, and foreign share of lending is $(1 - \bar{a} - sh\bar{a}re) = (1 - share)/(1+a)$. Since the GSEs share of borrowing is zero (the case where $G^- = 0$), share of domestic and foreign borrowing is as without GSEs and denoted by $share$ and $(1 - share)$ respectively.
2. In the initial guess of the steady-state probabilities that a borrower of overnight funds is borrowing from a domestic bank, γ^-_D, and that a lender of overnight funds is lending to a domestic bank, γ^+_D add the guess γ^-_G to equal the probability that a borrower is borrowing from a GSE.[32]

- step 2:

[32]For example, let the fraction of lending orders of a GSEs with equity 1 be 0.5, so that half of equity is put in as lending orders (this could be as large as 11, which is the total assets given banks capital constraint). Also, lets assume that the share of GSEs is 95%, so that 95% of total equity belongs to GSEs. From here we can calibrate the share of the mass of lending orders given by GSEs to be $\bar{a}G^+$=0.475.

1. Adjust the expected rates and expected liquidity cost and benefit of each sector given that if a lender is a GSE, its outside option is the ON RRP rate with the probability of such occurrence.

- step 3:

1. Add $G+$ to the total mass of lending so that $\bar{M}^+ = \bar{a}G^+ + sh\bar{a}reD^+ + (1 - sh\bar{a}re)F^+$.
2. Compute the probabilities that a borrower of overnight funds is borrowing from a domestic bank a foreign bank or a GSE.
3. Use these probabilities when computing the average federal funds rate, such that if you borrow form a GSE, its outside lending option is ON RRP.
4. The new market tightness equal to

$$\tilde{\theta} = M^- / \bar{M}^+$$

This means that the market clearing includes GSEs is

$$\gamma_{ss}^+(sh\bar{a}reD^+ + \bar{a}G^+ + (1 - \bar{a} - sh\bar{a}re)F^+) = \gamma_{ss}^-(shareD^- + (1 - share)F^-)$$

which equals

$$\frac{1}{1+a}\big(\gamma_{ss}^+(shareD^+ + (1-share)F^+ + aG^+)\big) = \gamma_{ss}^-\big(shareD^- + (1-share)F^-\big)$$

substitute this to E_{ss}^g, we get

$$E_{ss}^g = \beta[share\big(r_{ss}^D\bar{b}_{ss}^D + (r^{ior} - tax)\bar{m}_{ss}^D - r^d\bar{d}_{ss}^D - (r^{dw} - r^{ior} + tax)(1 - \gamma_{ss}^-)D^-$$
$$+ (r^{ff} - r^{ior} + tax)\gamma_{ss}^+D^+ - tax\gamma_{ss}^-D^-\big)$$
$$+ (1 - share)\big(r_{ss}^F\bar{b}_{ss}^F + r^{ior}\bar{m}_{ss}^F - r^d\bar{d}_{ss}^F - (r^{dw} - r^{ior})(1 - \gamma_{ss}^-)F^- + (r^{ff} - r^{ior})\gamma_{ss}^+F^+\big)$$
$$- \frac{\gamma_{ss}^+(r^{ff} - r^{ior})}{1+a}(shareD^+ + (1 - share)F^+ + aG^+)]$$

Update rule remains the same.

Table 16: Ratio of Reserves to Assets by Bank

FBO_NAME	FBO_COUNTRY	RCFD0090	RCFD2170	RtoA	DT
NA	NA	95430475	102106424	0.9346177	2011q1
DEUTSCHE BANK AKTIENGESELLSCHAFT	GERMANY	88710050	189404557	0.4683628	2011q3
DEUTSCHE BANK AKTIENGESELLSCHAFT	GERMANY	85218606	180621205	0.4718084	2011q4
NA	NA	76927001	83331189	0.9231478	2011q2
NA	NA	74549982	84977182	0.8772941	2010q1
DEUTSCHE BANK AKTIENGESELLSCHAFT	GERMANY	66694865	160950357	0.4143816	2011q2
NA	NA	63469643	74275331	0.8545185	2010q2
NA	NA	61864040	88753978	0.6970284	2010q3
NA	NA	61473961	70416011	0.8730111	2011q3
NA	NA	60463625	68898506	0.8775753	2010q4
CREDIT SUISSE AG	SWITZERLAND	53999477	77409493	0.6975821	2011q1
NA	NA	53728369	71098645	0.7556877	2011q3
CREDIT SUISSE AG	SWITZERLAND	51700101	75290396	0.6866759	2011q2
NA	NA	50193427	64808772	0.7744851	2009q4
CREDIT SUISSE AG	SWITZERLAND	43635101	69113088	0.6313580	2010q4
BANK OF NOVA SCOTIA, THE	CANADA	42737162	91658890	0.4662631	2011q4
NA	NA	42351554	63383994	0.6681743	2011q1
NA	NA	40920376	61507055	0.6652956	2011q2
DNB BANK ASA	NORWAY	40124843	41616323	0.9641612	2011q1
DEUTSCHE BANK AKTIENGESELLSCHAFT	GERMANY	39979503	134712772	0.2967759	2011q1
BANK OF NOVA SCOTIA, THE	CANADA	39726032	87261229	0.4552541	2011q3
NA	NA	39711447	46940914	0.8459879	2011q4
CREDIT SUISSE AG	SWITZERLAND	39403406	92246579	0.4271530	2011q3
CREDIT SUISSE AG	SWITZERLAND	38235101	86674671	0.4411335	2011q4
BANK OF NOVA SCOTIA, THE	CANADA	37902065	69819144	0.5428606	2011q2
NA	NA	35624720	50663545	0.7031628	2011q4
SVENSKA HANDELSBANKEN AB (PUBL)	SWEDEN	34681030	36892146	0.9400654	2011q4
CREDIT SUISSE AG	SWITZERLAND	34553101	35036520	0.9862024	2009q1
SVENSKA HANDELSBANKEN AB (PUBL)	SWEDEN	34416029	37297427	0.9227454	2011q3
BNP PARIBAS	FRANCE	33392111	74354368	0.4490941	2011q4
SOCIETE GENERALE	FRANCE	33128293	84665720	0.3912834	2011q3
CREDIT SUISSE AG	SWITZERLAND	32730101	36849425	0.8882120	2008q4
NA	NA	31900601	54900562	0.5810615	2010q3
MIZUHO BANK, LTD.	JAPAN	30556779	82006799	0.3726128	2011q4
BANK OF NOVA SCOTIA, THE	CANADA	28815342	42134679	0.6838866	2010q1
MUFG BANK, LTD.	JAPAN	28534306	95708971	0.2981362	2011q3
BANK OF NOVA SCOTIA, THE	CANADA	27254385	56891803	0.4790564	2011q1
CANADIAN IMPERIAL BANK OF COMMERCE	CANADA	27150005	30619092	0.8867018	2011q1
SUMITOMO MITSUI BANKING CORPORATION	JAPAN	27029299	69659091	0.3880226	2011q3
ABBEY NATIONAL TREASURY SERVICES PLC	UNITED KINGDOM	26931695	27308914	0.9861870	2011q2